Example relations problems (Katherine E. Stange, Math 2001, CU Boulder)

1. Draw an arrow diagram of the relation $R = \{(1/2, 3/4), (4/5, 3/4), (1/3, 1/2)\}$ on the set $A = \{1/2, 1/3, 1/4, 3/4, 4/5\}$.

2. Give the ordered pairs notation and the arrow diagram for the relation $\subseteq$ on $A = \{\emptyset, \{3, 4\}, \{1, 2\}, \{2\}, \{3\}\}$.

3. Give an example of a relation on the set $A = \{e\}$. Give it as a set of ordered pairs, or as an arrow diagram (your choice).

4. Now give another, different, example of a relation on the set $A = \{e\}$.

5. For each of the following relations, determine if it is reflexive, symmetric, transitive and/or equivalence.

   (a) The relation $\leq$ on $\mathbb{Z}$.
   - Reflexive? YES / NO
   - Symmetric? YES / NO
   - Transitive? YES / NO
   - Equivalence? YES / NO

   (b) The relation $\neq$ on $\{0\}$.
   - Reflexive? YES / NO
   - Symmetric? YES / NO
   - Transitive? YES / NO
   - Equivalence? YES / NO

   (c) The relation $\{(e, e)\}$ on $\{e\}$.
   - Reflexive? YES / NO
   - Symmetric? YES / NO
   - Transitive? YES / NO
   - Equivalence? YES / NO

   (d) The relation $\{(a, a), (b, b), (a, b), (b, a)\}$ on $\{a, b, c, d\}$.
   - Reflexive? YES / NO
   - Symmetric? YES / NO
   - Transitive? YES / NO
   - Equivalence? YES / NO

6. Give a relation on the set $\{a, b, c, d\}$ which is not reflexive and not symmetric. (You can give an arrow diagram or a set of ordered pairs.)

7. What is the equivalence class of $\{1\}$ under the equivalence relation "has the same cardinality as" on the set $\mathscr{P}(\{1, 2, 3\})$?

8. Draw an arrow diagram of the relation $R = \{(3, 10), (4, 3), (3, 2)\}$ on the set $A = \{2, 3, 4, 10\}$.

9. Give the ordered pairs notation and the arrow diagram for the relation $\equiv$ (mod 5) (in english, *equivalence modulo 5*) on $A = \{14, 15, 16, 17, 18, 19, 20\}$.

10. Give an example of a relation on the set $A = \{e, f\}$. Give it as a set of ordered pairs ***and*** as an arrow diagram.

11. For each of the following relations, determine if it is reflexive, symmetric, transitive and/or equivalence.

    (a) The relation $\geq$ on $\mathbb{R}$.
        • Reflexive? YES / NO
        • Symmetric? YES / NO
        • Transitive? YES / NO
        • Equivalence? YES / NO

    (b) The relation $\neq$ on $\mathbb{Z}$.
        • Reflexive? YES / NO
        • Symmetric? YES / NO
        • Transitive? YES / NO
        • Equivalence? YES / NO

    (c) The relation $\{(e, e), (f, e), (e, f)\}$ on $\{e, f\}$.
        • Reflexive? YES / NO
        • Symmetric? YES / NO
        • Transitive? YES / NO
        • Equivalence? YES / NO

(d) The relation *sends 0 to the same place* on the set of all functions from $\mathbb{R}$ to $\mathbb{R}$.

- Reflexive? YES / NO
- Symmetric? YES / NO
- Transitive? YES / NO
- Equivalence? YES / NO

12. Give a relation on the set $\{a, b, c\}$ which is reflexive and not symmetric. (You can give an arrow diagram or a set of ordered pairs; your choice.)

13. What is the equivalence class of 2 under the equivalence relation *has the same last digit* on the set $\{x \in \mathbb{Z} : 0 \le x \le 50\}$?

14. How many different equivalence classes are there in total, for the equivalence relation of the last question?

15. Give an example of an equivalence relation with equivalence classes whose cardinalities are not all the same. Give an example of two classes with different cardinalities.

16. How many different equivalence relations are possible on a set of 3 elements? Why?

17. Suppose A has cardinality n, and suppose there is an equivalence relation on A whose m equivalence classes all have the same cardinality. What size are the equivalence classes?

18. If R and S are two equivalence relations on a set A, is $R \cup S$ an equivalence relation on A? Why or why not?

19. How many different partitions are there of a two-element set?

20. How many different equivalence relations are there on a two-element set?

21. Give an example of a relation on $\mathbb{Z}$ which is not transitive.