Example logic problems (Katherine E. Stange, Math 2001, CU Boulder)

1. Give the truth table for the expression $P \implies Q$.

2. Give the truth table for the expression $(P \vee Q) \implies (\sim R)$. For partial credit, show your intermediate columns (e.g. $\sim R$). Note: this has 3 variables, so it should have 8 rows.

3. Give an example of a boolean expression in two variables P and Q which is a *tautology*, in other words, in its truth table, it evaluates to T (true) for all inputs. Note: the expression must involve both variables somehow.

4. Give an example of a boolean expression in two variables P and Q which is a *contradiction*, in other words, in its truth table, it evaluates to F (false) for all inputs. Note: the expression must involve both variables somehow.

5. For each statement, give the converse and the contrapositive. It is important that the sentences you write make english grammatical sense (e.g. don't use a variable before introducing it).

   (a) If $x \in \mathbb{Z}$ is prime, then x is odd.

   - Converse:

   - Contrapositive:

(b) If $X \subseteq S$, then $|X| = 3$ or $|X| = 2$.

- Converse:

- Contrapositive:

(c) If the graph G has n vertices, then it has at least one edge.

- Converse:

- Contrapositive:

6. For each symbolic statement, determine if it is true or false.

(a) $\forall x \in \mathbb{Z}, x > 3$.

TRUE, FALSE

(b) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x = -y$.

TRUE, FALSE

(c) $\exists x \in \mathbb{Q}, x^2 = 2$.

TRUE, FALSE

(d) $\exists X \subseteq \mathbb{R}, 3 \in X$.

TRUE, FALSE

7. For each english statement, give the same statement in symbolic notation (**no english words**), including such things as quantifiers ($\exists$, $\forall$), boolean operators ($\sim$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$), etc.

(a) The number a is an even integer.

(b) If $x \in \mathbb{Z}$, then x is even.

(c) Every subset of S has cardinality less than 6.

(d) All rational numbers have rational squares.

8. Negate each of the following statements. Simplify your negations to a reasonable level for full credit (that is, don't just give "It is false that X" as the negation of X; this is correct but not enlightening; try to "move the negation in" as far as you can, at the very least inside the quantifier.)

   (a) There are no polynomials of degree 7.

   (b) All polynomials of degree 6 have exactly 6 roots in the complex numbers.

   (c) Among polynomials f with integer coefficients, if f has degree 3, then f^2 has degree 6.

   (d) There are polynomials having degree 3 and no rational roots.

   (e) Every polynomial has at least one root.

   (f) If a polynomial has a root, then it has degree at least 1.

9. True or False

   (a) $P \Rightarrow Q = P \vee (\sim Q)$

   (b) $\sim (P \vee Q) = (\sim P) \vee (\sim Q)$

   (c) $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$

10. Fill in the missing symbols in the blanks (choose from $\Leftrightarrow$, $\Rightarrow$, $\wedge$ and $\vee$):

   (a) $\sim (P \Rightarrow Q) = P$____$(\sim Q)$

   (b) $\sim (P \wedge Q) = (\sim P)$____$(\sim Q)$

   (c) $P \wedge (Q \vee R) = (P$____$Q)$____$(P$____$R)$

   (d) $P \Leftrightarrow Q = (P \Rightarrow Q)$____$(Q \Rightarrow P)$

11. Draw lines connecting expressions in the first column to their negations in the second column. Everything has a pair.

$P \Rightarrow Q$	$(\sim P) \wedge (\sim Q)$
$(\sim P) \Rightarrow Q$	$P \wedge (\sim Q)$
$\sim (P \Leftrightarrow Q)$	$(\sim P) \vee (\sim Q)$
$P \wedge Q$	$(\sim P) \Leftrightarrow (\sim Q)$
$P \wedge (\sim Q)$	$(\sim P) \vee Q$

12. Give a logically equivalent expression for each, **using no negation symbols**:

(a) $\sim((\sim P) \wedge (\sim Q))$

(b) $(\sim P) \vee Q$

(c) $(\sim P) \Leftrightarrow (\sim Q)$

(d) $(\sim P) \Rightarrow (\sim Q)$