Example counting problems (Katherine E. Stange, Math 2001, CU Boulder)

1. Suppose your turn in a game depends on rolling one six-sided die and flipping one two-sided coin. How many outcomes are possible?

2. There are six different vegetables on the counter. How many ways can you select one to use for a main course, and one to use for a dessert?

3. A doctor must visit 4 different patients A, B, C and D, in some order. In addition, for each patient, she must order or not order a test. How many ways can this happen? In other words, how many ways can she visit the patients in some order and decide on tests for each. (For example, one outcome is (1st visit C, test; 2nd visit B, no test; 3rd visit D, no test; 4th visit A, test).)

4. You are required to choose a password for a virtual hamster racing website (we all need something to keep us busy in the age of coronavirus, and we can't do it in reality anymore). The password must be 8 characters long. Seven of the characters are letters (26 letters to choose from), and one character is a 'special character' (meaning chosen from the 10 shown above the numbers on your keyboard). The special character can go anywhere among the others.

5. Consider a combination lock which uses the digits 0 through 9, where a combination involves exactly 4 digits, and the lock has the restriction that no two *consecutive* digits in a valid combination can be the same. (For example, 3346 is not allowed but 3463 is ok.) How many valid combinations are there?

6. How many ways can you break a group of 13 racing hamsters up into *three* disjoint sets (empty sets are ok), called *Awesome Team*, *Mediocre Team* and *Tasty Team*? In other words, how many ways can you partition a set of 13 things into 3 disjoint differently-named subsets whose union is the full set?

7. How many ways can you make a 7-letter word by rearranging the letters IEEPVVV?

8. How many ways can you choose a subset of $\{a, b, c, d, e, f, g\}$, where the subset must have size 3 or 5, and must contain the element f?

9. You must pair up everyone in the class for a partner activity. There are 8 students. How many ways can the pairs be formed? Order doesn't matter (within pairs or among pairs).

10. You roll two dice. How many different outcomes are there? Hint: The dice are identical, so the results are not ordered, i.e. (5,6) is the same as (6,5), but don't forget about doubles like (5,5)!

11. How many n-digit binary sequences (sequences of 0s and 1s) contain exactly k 1s, but do not have a 1 in the first position?