# Proof For Feedback for Apr 14

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**Theorem 1.** *Define the following relation on $\mathbb{Z}$: $xRy$ if $x = 2^k y$ for some $k \in \mathbb{Z}$. Then this relation is an equivalence relation.*

*Hint: Do some examples to make sure you understand the definition. For example, $2R1$ but $3$ is not related to $5$.*

*Proof.* We need to check three properties.

**Reflexivity:** Let $x \in \mathbb{Z}$. Then $x = 2^0 x$, so that $xRx$.

**Symmetry:** Let $x, y \in \mathbb{Z}$, and suppose $xRy$. Then $x = 2^k y$ for some $k \in \mathbb{Z}$. Therefore $y = 2^{-k} x$, so $yRx$.

**Transitivity:** Let $x, y, z \in \mathbb{Z}$, and suppose $xRy$ and $yRz$. Then $x = 2^k y$ and $y = 2^\ell z$. Combining these, $x = 2^k 2^\ell z = 2^{k+\ell} z$, so that $xRz$. $\square$