Counting Quiz (Katherine E. Stange, Math 2001, Spring 2023, CU Boulder)

Name:

For each counting problem, please determine the answer but preferably leave it unsimplified form. You may use binomial coefficients and factorials in your final answer if appropriate. Provide one or two sentences or phrases/point-form of brief justification.

If the questions are unclear, please ask during the test and I will clarify.

1. You are in charge of a restaurant and must set the menu for the week. Each weekday you must serve either tofu or pineapple for the main course, and either chocolate, sardines or okra for dessert. How many ways can you set the menu for the week (7 days)?

    (a) Numerical answer (unsimplified):

    (b) Justification (one or two sentences):

2. You have 5 **different** hamsters. You must choose a subset of hamsters to give to your mother. You don't want her to be sad, so you must give her at least one of the hamsters. How many ways can this be done?

    (a) Numerical answer (unsimplified):

    (b) Justification (one or two sentences):

3. You must lay out 7 **identical** cups on the table in a row. Exactly three of them must be right-side-up and the rest must be upside-down. How many different ways can this be done? (Examples: one solution is Up-Up-Down-Down-Down-Up-Down. But Up-Up-Up-Down-Down-Down-Up is not a valid solution because too many of them are up.)

    (a) Numerical answer (unsimplified):

    (b) Justification (one or two sentences):

4. How many ways can you arrange 9 pencils in a row, where 3 are identical blue pencils, 3 are identical green pencils, and 3 are identical red pencils?

    (a) Numerical answer (unsimplified):

    (b) Justification (one or two sentences):

5. Let $X = \{a, b, c, d, e, f, g\}$. How many ways can you choose a subset of size 4 or 5 that contains a but does not contain g?

    (a) Numerical answer (unsimplified):

    (b) Justification (one or two sentences):

6. You have 19 **different** gifts. How can you distribute them among three friends (Ali, Bob and Cal), so that everyone gets at least one object?

    (a) Numerical answer (unsimplified):

    (b) Justification (one or two sentences):