

## 1 Assignment

Prove that the number of ways to order  $n$  items in a row is  $n!$ .

*Hint: Use the multiplication principle we discussed in class. Practice writing it very nicely.*

## 2 A possible solution

We can order  $n$  items in a row by choosing the first item, then the second, then the third, etc., placing them in a row as we choose them.

As there are  $n$  items when we begin, there are  $n$  ways to choose the first item.

There are then  $n - 1$  ways to choose the second item (since one possibility has been removed).

There are then  $n - 2$  ways to choose the third item.

Continuing in this way for all  $n$  items, we eventually find that there are 2 ways to choose the second-to-last item and only 1 way to choose the last item.

Combining all these steps by the multiplication principle, we have

$$n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot 3 \cdot 2 \cdot 1 = n!$$

ways to order all  $n$  items in a row.

## 3 ChatGPT

This can be proven using the fundamental counting principle. If we have  $n$  items, we can choose the first item in  $n$  different ways, the second item in  $n-1$  different ways, the third item in  $n-2$  different ways, and so on, until we choose the  $n$ th item in 1 way.

The number of ways to order the  $n$  items is given by the product of the number of choices for each item:  $n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 2 \cdot 1 = n!$ .

This is the definition of  $n$  factorial, denoted by  $n!$ . It represents the total number of ways to order  $n$  items, and it grows rapidly as  $n$  increases. For example,  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ , which means there are 120 different ways to order 5 items.

*Comments on ChatGPT: the last paragraph isn't necessary; it is not part of the proof, it's just some surrounding information/background. Otherwise, this is not bad.*