Math 2001, Spring 2023. Katherine E. Stange.

1 Assignment

Prove the following theorem.

Theorem 1. *There are no integers a and b so that $4a^2 - b^2 = 1$.*

Hint: proof by contradiction. one method is to try factoring the left side; which integers can multiply to 1?

Proof. Suppose, for a contradiction, that $4a^2 - b^2 = 1$ for some integers a and b. Let us factor the left side of the equation:

$$(2a - b)(2a + b) = 1.$$

Then $2a + b$ and $2a - b$ are integers whose product is 1. Therefore, we have one of the following two cases:

Case I: $2a + b = 2a - b = 1$**.** In this case, $b = 0$ and $2a = 1$, so $a = 1/2$, which is not an integer; contradiction.

Case II: $2a + b = 2a - b = -1$**.** In this case, $b = 0$ and $2a = -1$, so $a = -1/2$, which is not an integer; contradiction.

In both cases we have reached a contradiction, proving the theorem. □