Math 2001, Spring 2023. Katherine E. Stange.

1 Assignment

Prove the following theorem **by contradiction**.

Theorem 1. *Let $a, b, c \in \mathbb{Q}$. There is at most one integer x which satisfies $ax + b = c$.*

Note: this is practice in writing a proof by contradiction. An alternate method would be to solve for x, but I want you to avoid that method and instead try to do this by contradiction.

2 Solution

The statement had an error!! Some of you probably discovered this while writing a proof. Here's the corrected statement and proof.

Theorem 2. *Let $a, b, c \in \mathbb{Q}$, with $a \neq 0$. There is at most one integer x which satisfies $ax + b = c$.*

Proof. Let $a, b, c \in \mathbb{Q}$. Suppose for a contradiction that there are two distinct integers x_1 and x_2 such that $ax_1 + b = c$ and $ax_2 + b = c$. Then

$$ax_1 + b = c = ax_2 + b.$$

Therefore, subtracting b from both sides,

$$ax_1 = ax_2.$$

Now, dividing by a (which is non-zero),

$$x_1 = x_2.$$

This contradicts the assumption that x_1 and x_2 are distinct. □