

1 Assignment

Prove the following theorem.

Theorem 1. *Suppose $x \in \mathbb{Q}$ and $y \notin \mathbb{Q}$. Then $y - x \notin \mathbb{Q}$.*

Hint: proof by contradiction (see recent class notes, or Hammack, Chapter 6). Reach out to me for hints if you are stuck!

2 An example solution

Proof. Let $x \in \mathbb{Q}$ and let $y \notin \mathbb{Q}$. Let us suppose for a contradiction that $y - x \in \mathbb{Q}$. Then, by the definition of the rational numbers, we may write

$$x = \frac{a}{b}, \quad y - x = \frac{c}{d}$$

where $a, b, c, d \in \mathbb{Z}$. Therefore

$$y = (y - x) + x = \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}.$$

But this is a ratio of integers, hence $y \in \mathbb{Q}$, a contradiction. □