Quantum Gates Exercise – Math 4440

1. Here's a new gate called the square root of not (presented in the $|0\rangle$, $|1\rangle$ basis as usual):
$$\sqrt{X} = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$$
Do the following:
   (a) Check that this gate is unitary.
   (b) Check that it squares to the Pauli X or NOT gate.
   (c) Figure out what $\sqrt{X}\,|0\rangle$ is in terms of $|0\rangle$ and $|1\rangle$.
   (d) Figure out what $\sqrt{X}\,|1\rangle$ is in terms of $|0\rangle$ and $|1\rangle$.
2. Verify by direct computation that $R_{x,\pi/2}\,|1\rangle = -i\,|i\rangle$.
3. Determine a 2-qubit quantum circuit that will, on input $|00\rangle$, produce output
$$\frac{1}{2}\,|00\rangle + \frac{1}{2}\,|01\rangle + \frac{1}{2}\,|10\rangle + \frac{1}{2}\,|11\rangle\,.$$
4. Define a 3-qubit gate that can be used to compute a reversible form of OR.