Quantum Entanglement Exercise – Math 4440

Here is a 2-qubit state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|11\rangle.$$

1. Suppose you measure the first qubit and obtain a measurement of $|0\rangle$.
   (a) What is the state after this measurement?
   (b) If you now measure the second qubit, what are the possible results and probabilities of those results?
2. Suppose instead you measure the first qubit and obtain a measurement of $|1\rangle$.
   (a) What is the state after this measurement?
   (b) If you now measure the second qubit, what are the possible results and probabilities of those results?
3. Is this an entangled state? Justify.

Suppose you have $n+1$ qubits. We will write $|\vec{x}\rangle$ to mean the n-qubit classical state given by the number x in binary. So for example, if $n = 2$ then $\left|\vec{0}\right\rangle = |00\rangle$, $\left|\vec{3}\right\rangle = |11\rangle$ etc. Suppose the qubits are in this state:

$$|\Phi\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |\vec{x}\rangle |x \mod 2\rangle$$

Note: the notation $|a\rangle|b\rangle$ just means the concatentation of the two states (so maybe a is the state of the first qubit and b is the state of the second). In this context it helps to keep the two parts visually separate.

1. What is the resulting state if we measure the last qubit and obtain $|0\rangle$? Make sure you have normalized your state.
2. What is the resulting state if we measure the last qubit and obtain $|1\rangle$? Make sure you have normalized your state.