

Induction

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1 The induction format

Induction breaks one proof into two smaller proofs. For each theorem, write the **statement of** the base case and the inductive step. Don't do the proofs, just tell me what you need to prove for each of the two pieces. In fact, some of the statements are wrong!

1. Every positive integer is even or odd.
 - (a) **Base Case:** The number 1 is even or odd.
 - (b) **Inductive Step:** If k is even or odd, then $k + 1$ is even or odd.
2. Any non-negative integer can be written as a sum of four squares.
 - (a) **Base Case:**
 - (b) **Inductive Step:**
3. Any graph is Stygian.
 - (a) **Base Case:**
 - (b) **Inductive Step:**
4. Any equivalence relation is scary.
 - (a) **Base Case:**
 - (b) **Inductive Step:**

2 Induction proofs

Theorem 1. *Any number $n \geq 4$ of cents can be obtained by using 2 and 5 cent coins.*

A *Hamiltonian cycle* in a graph G is a cycle that includes every vertex of G exactly once.

Theorem 2. *Any hypercube has a Hamiltonian cycle.*

Theorem 3. *Let n be a positive integer, and consider a $2^n \times 2^n$ chessboard. For any square on the board which is removed, there is a tiling by 3-square-L's (drawn at left) of the remaining $4^n - 1$ squares.*