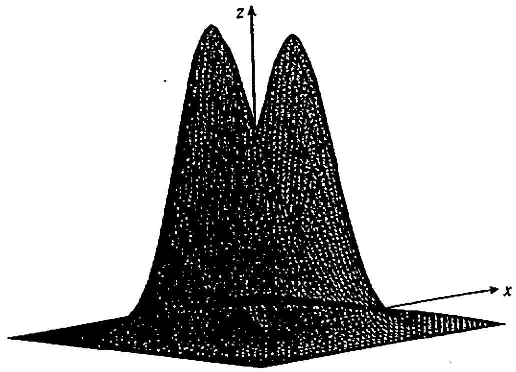
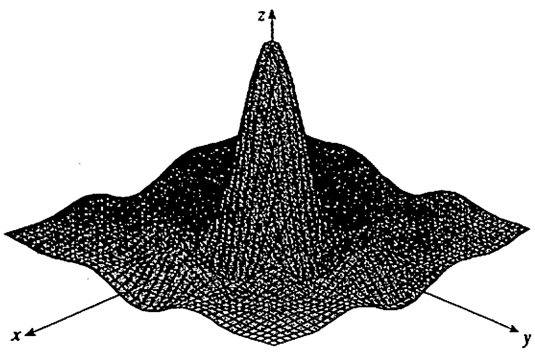
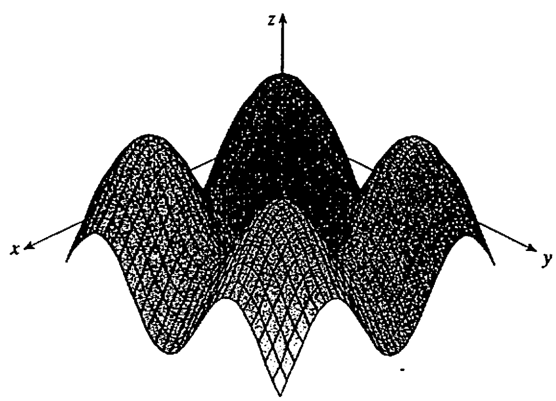


$z = (x^2 + 3y^2)e^{-x^2-y^2}$

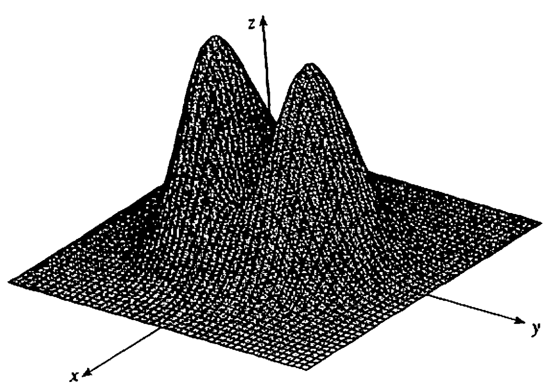
Surfaces



$$z = \frac{\sin x \sin y}{xy}$$



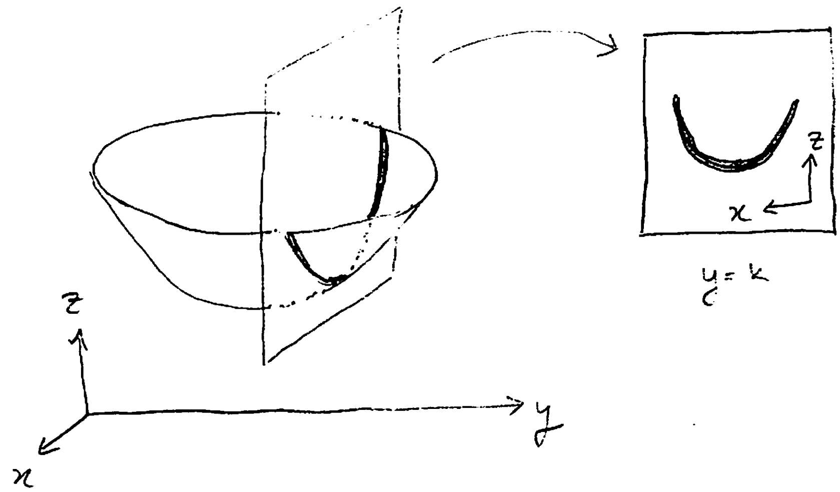
$z = \sin x + \sin y$



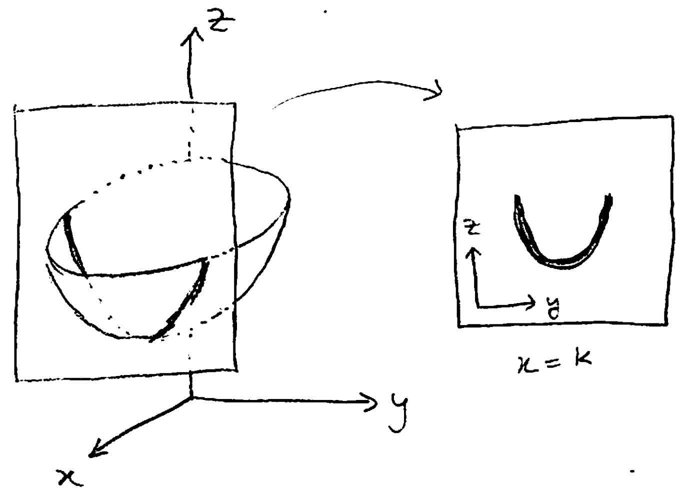
$z = (x^2 + 3y^2)e^{-x^2-y^2}$

Traces (or cross sections)

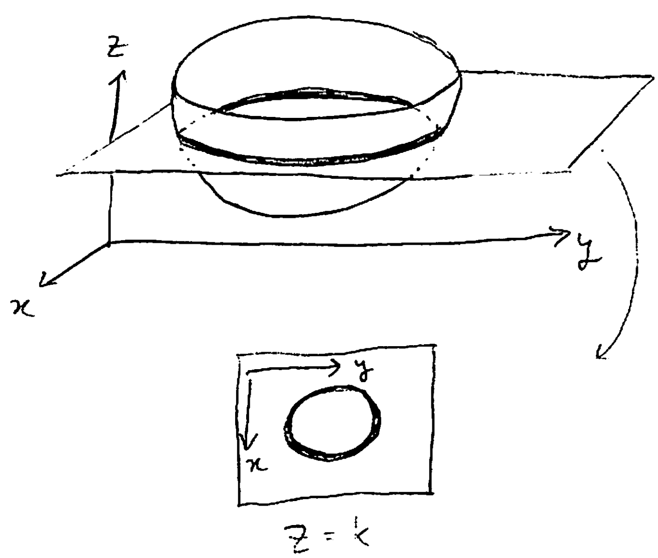
trace in $y=k$



trace in $x=k$



trace in $z=k$



Some examples:

1) vertical hollow cylinder

$z=k$ traces:

$x=k$ traces:

$y=k$ traces:

2) sphere (hollow)

$z=k$ traces:

$x=k$ traces:

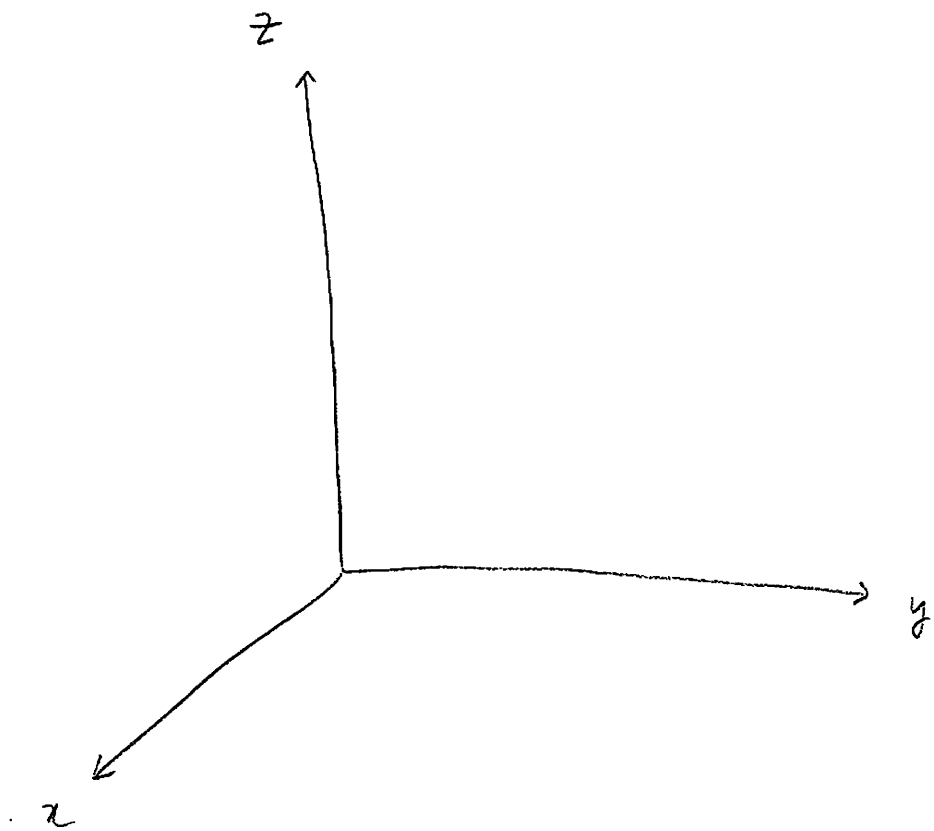
$y=k$ traces:

Sketch $y = x^2$ (in 3D)

$z = k$ traces

$y = k$ traces

$x = k$ traces

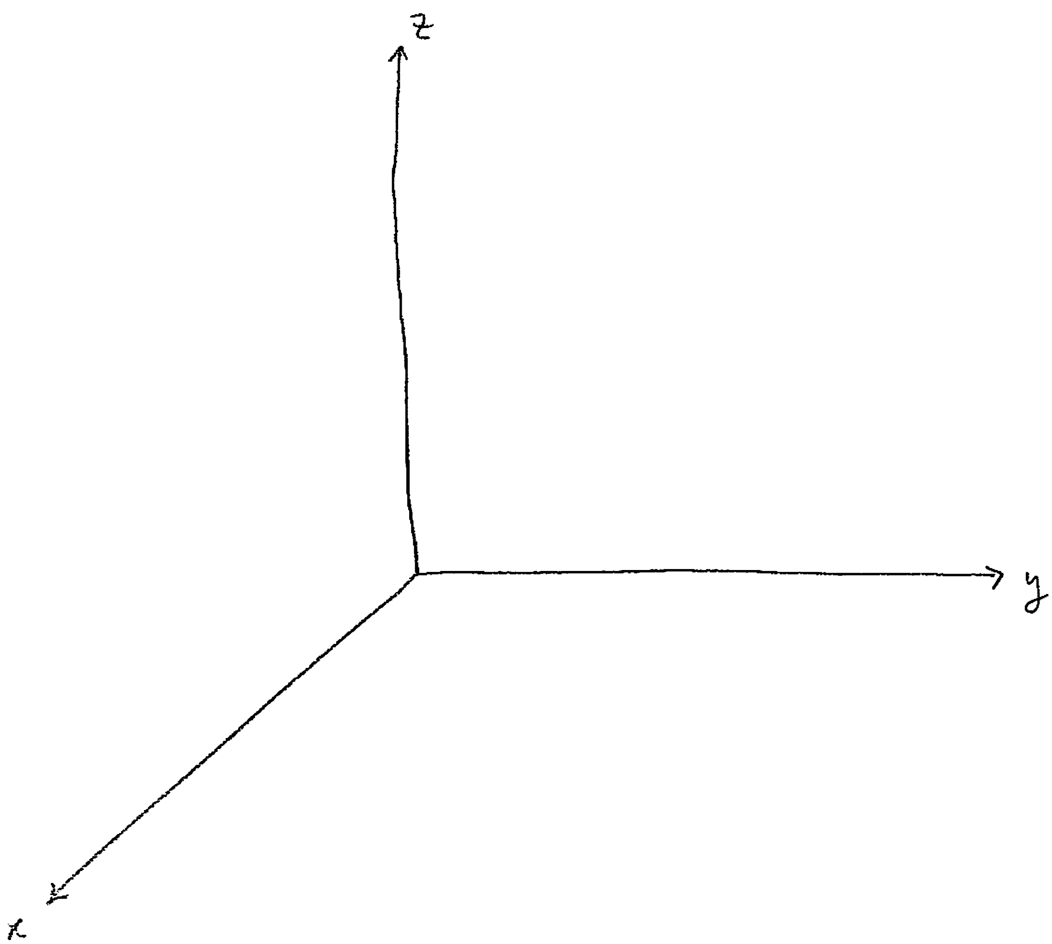


Example: Sketch $\frac{x^2}{9} + z^2 = 1$

$y = k$ traces

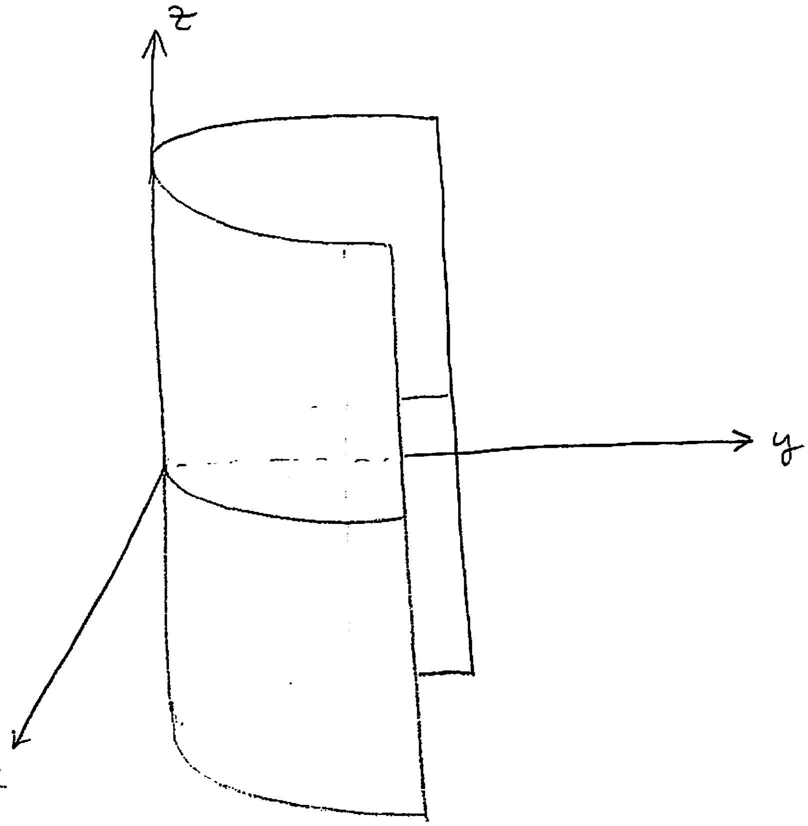
$z = k$ traces

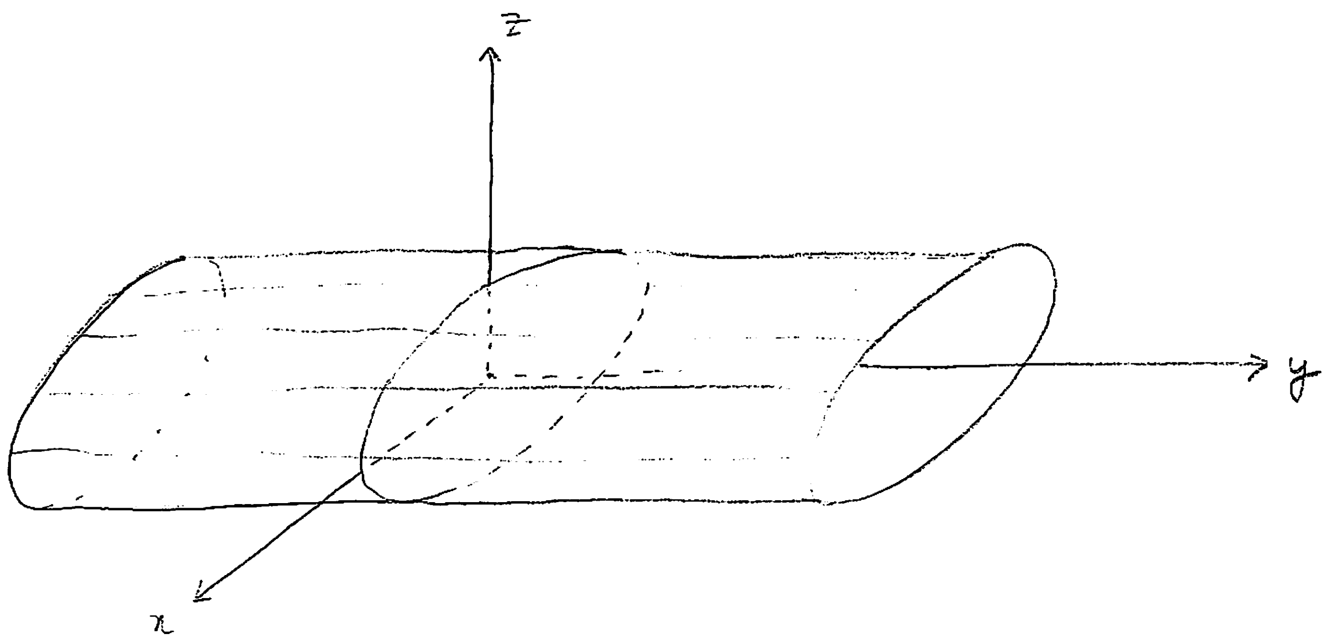
$x = k$ traces



<u>Solutions</u>

$y = x^2$

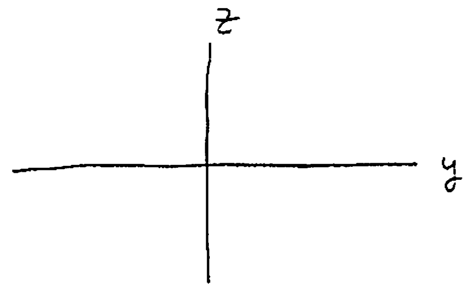




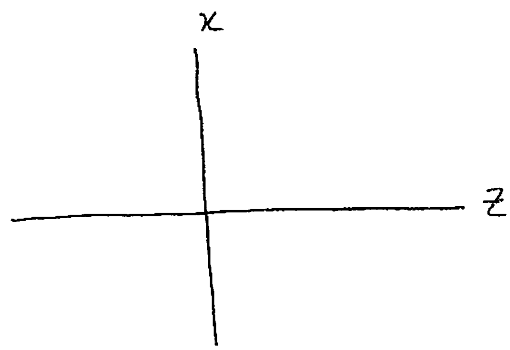
$$\frac{x^2}{9} + z^2 = 1$$

Example: use traces to sketch $x = y^2 + 9z^2$

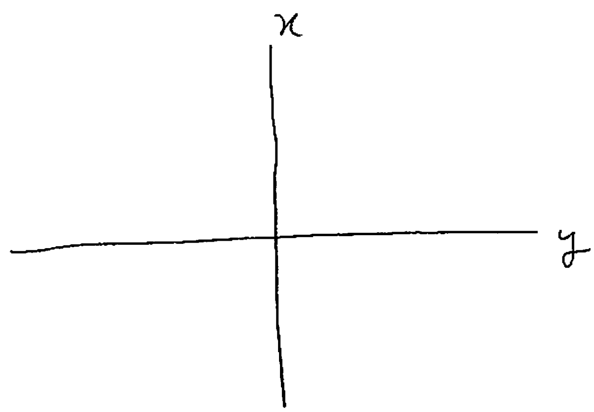
$x=k$ traces



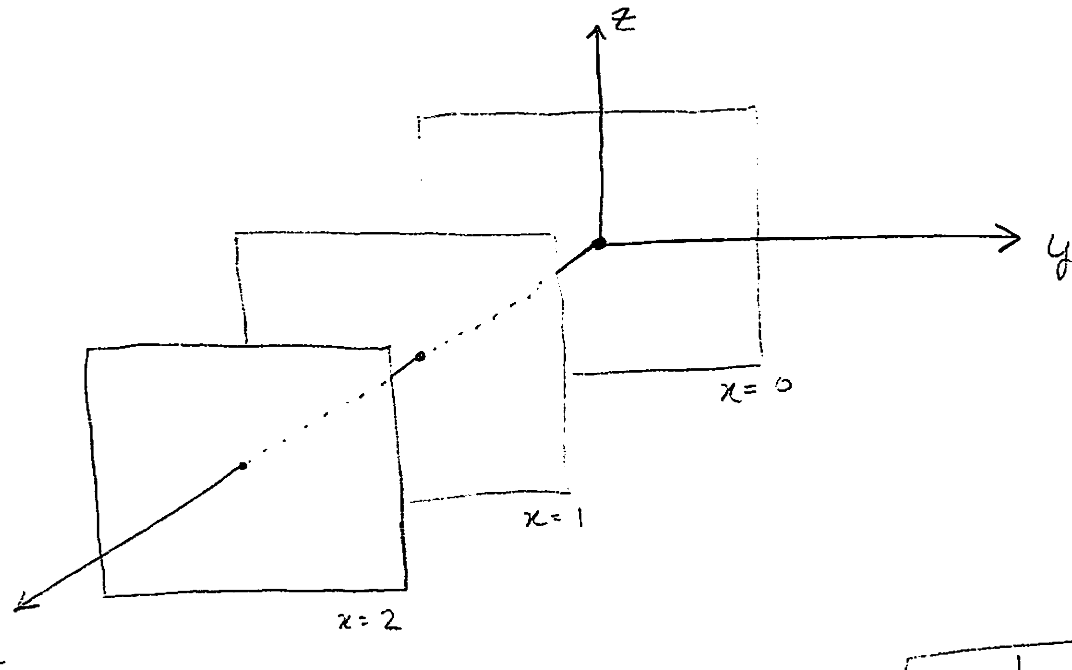
$y=k$ traces



$z=k$ traces

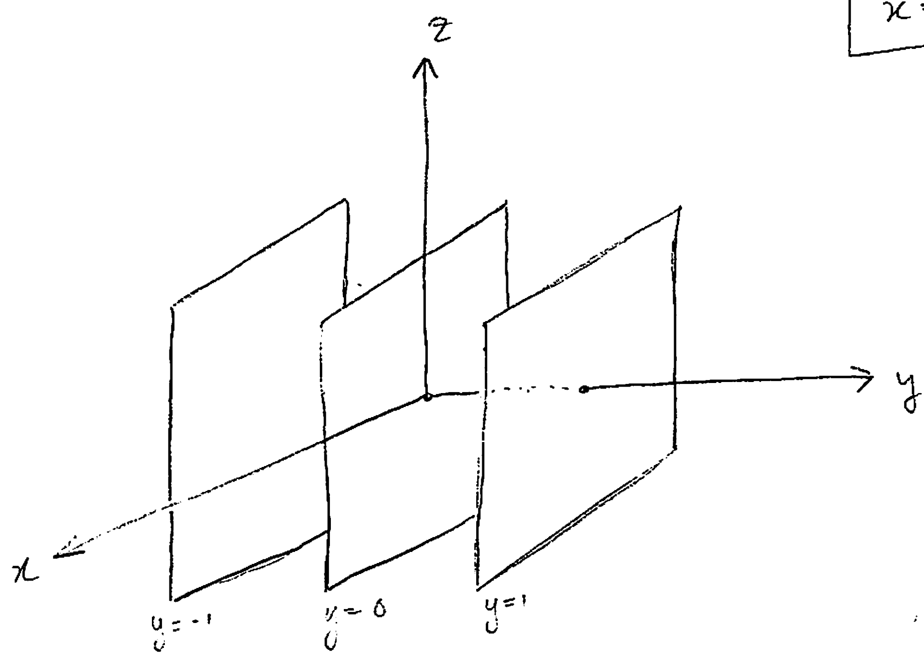


$x = k$ traces

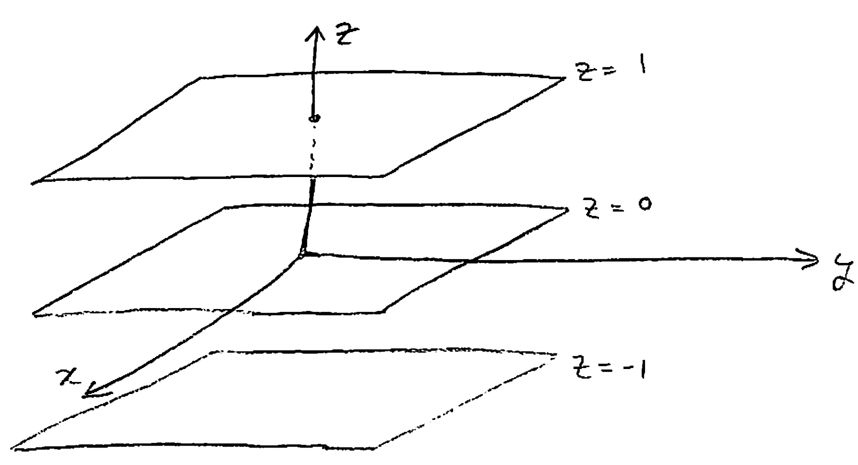


graphing
$x = y^2 + 9z^2$

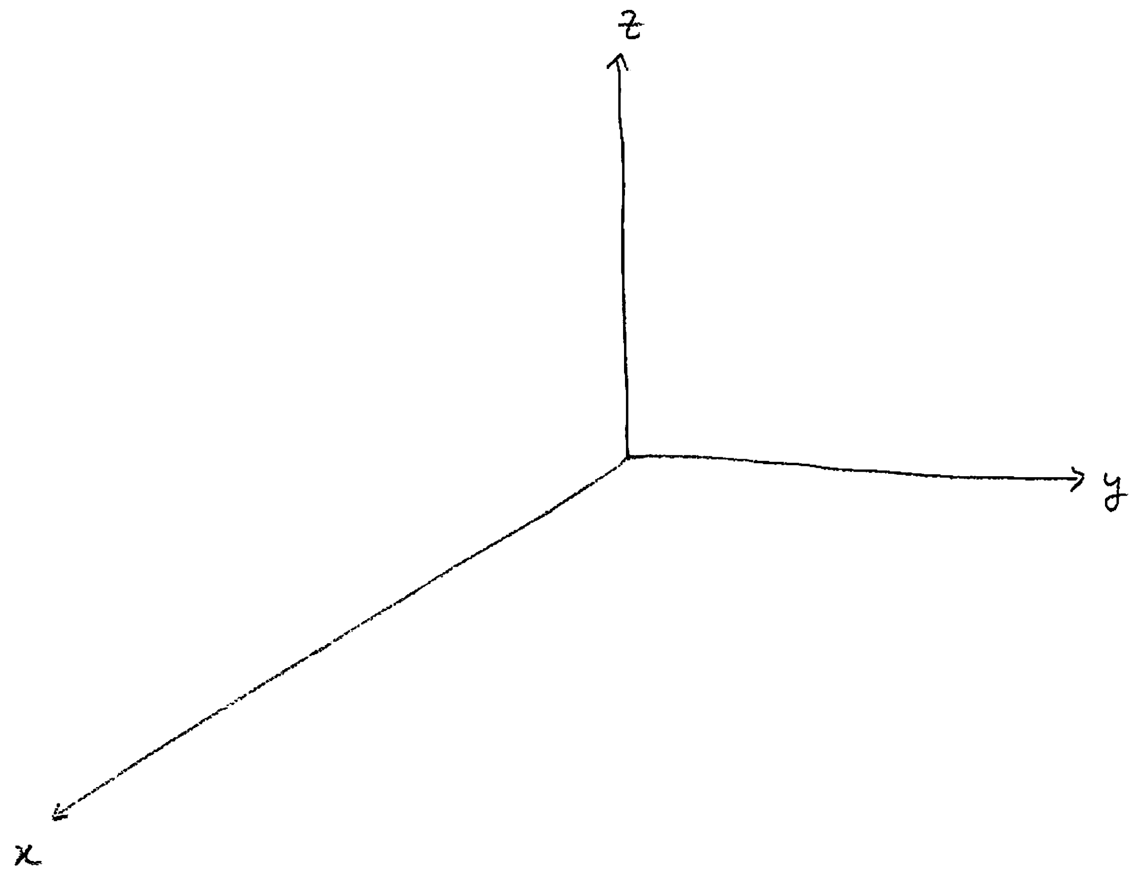
$y = k$ traces



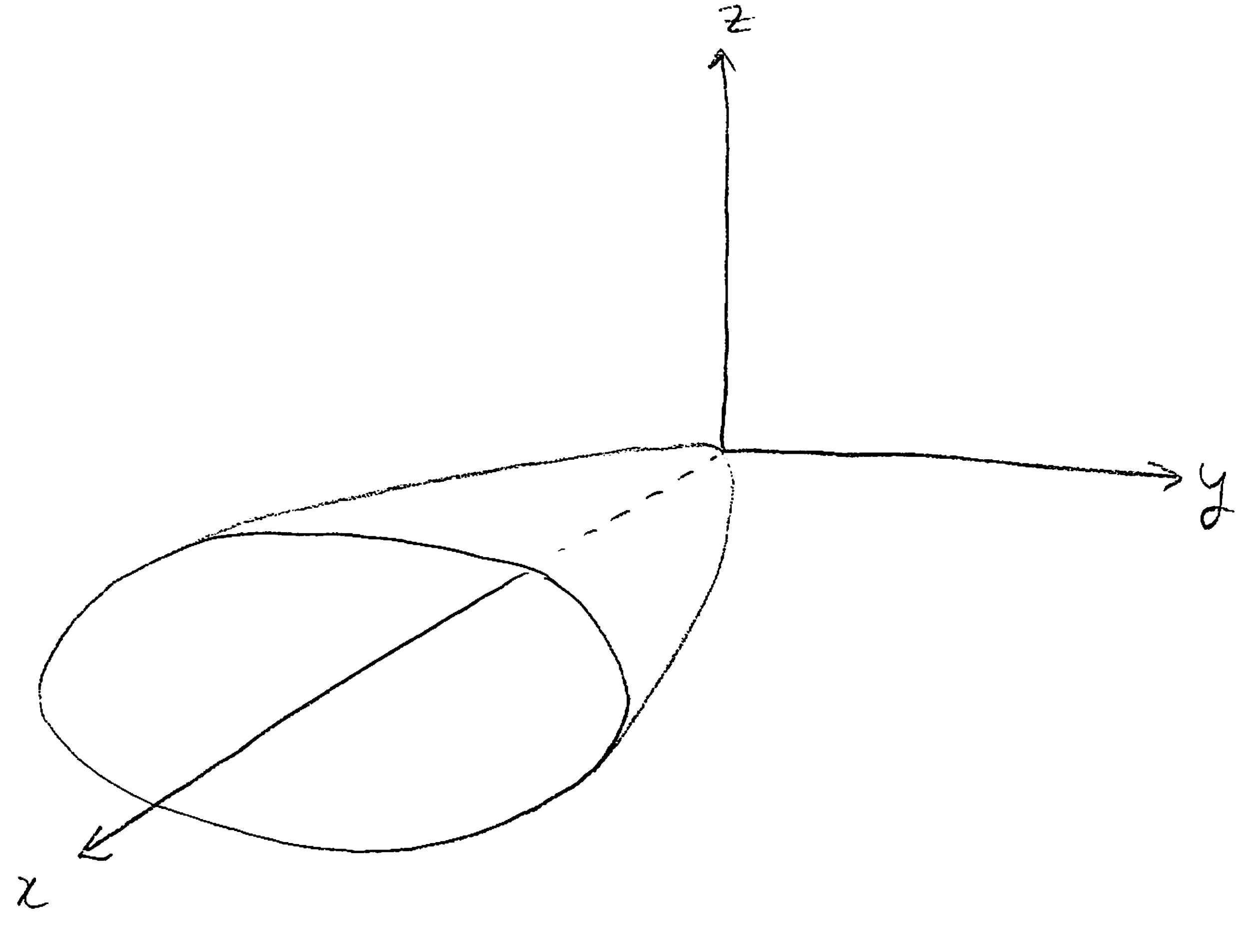
$z = k$ traces



Give it a shot! $x = y^2 + 9z^2$

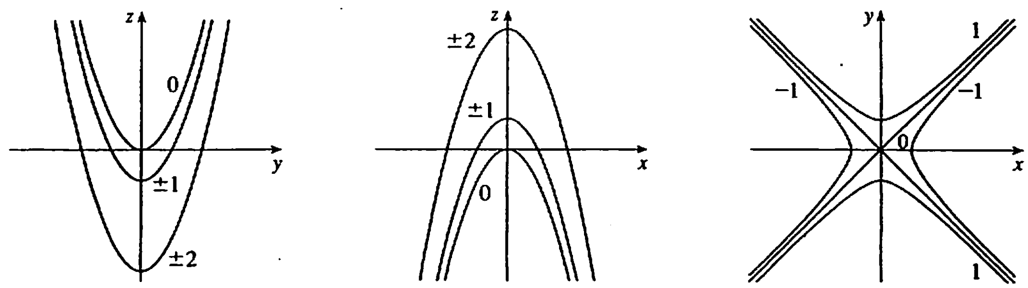


$x = y^2 + 9z^2$



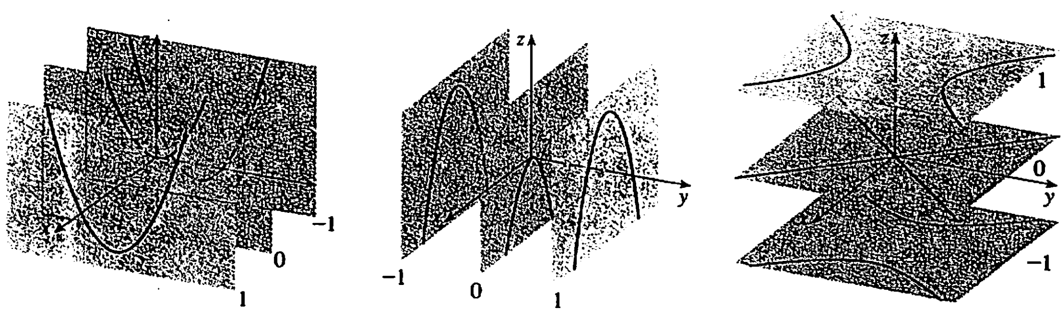
EXAMPLE 5 Sketch the surface $z = y^2 - x^2$.

SOLUTION The traces in the vertical planes $x = k$ are the parabolas $z = y^2 - k^2$, which open upward. The traces in $y = k$ are the parabolas $z = -x^2 + k^2$, which open downward. The horizontal traces are $y^2 - x^2 = k$, a family of hyperbolas. We draw the families of traces in Figure 6, and we show how the traces appear when placed in their correct planes in Figure 7.



Traces in $x = k$ are $z = y^2 - k^2$ | Traces in $y = k$ are $z = -x^2 + k^2$ | Traces in $z = k$ are $y^2 - x^2 = k$

FIGURE 6
Vertical traces are parabolas; horizontal traces are hyperbolas. All traces are labeled with the value of k.



Traces in $x = k$ | Traces in $y = k$ | Traces in $z = k$

FIGURE 7
Traces moved to their correct planes

In Module 13.6A you can investigate how traces determine the shape of a surface.

In Figure 8 we fit together the traces from Figure 7 to form the surface $z = y^2 - x^2$, a **hyperbolic paraboloid**. Notice that the shape of the surface near the origin resembles that of a saddle. This surface will be investigated further in Section 15.7 when we discuss saddle points.

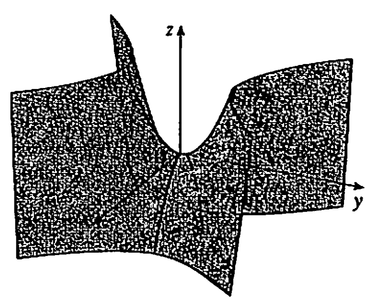


FIGURE 8
The surface $z = y^2 - x^2$ is a hyperbolic paraboloid.

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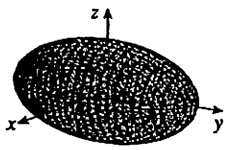
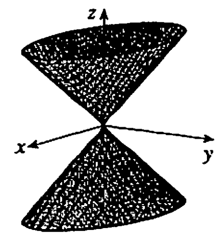
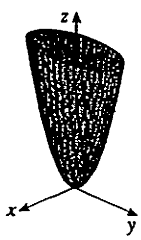
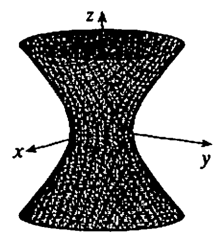
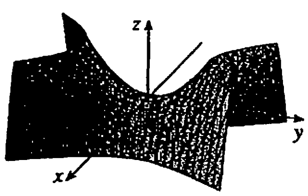
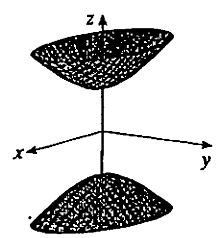
TABLE 1 Graphs of quadric surfaces

Surface	Equation	Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.	Cone 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.
Elliptic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	Hyperboloid of One Sheet 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.	Hyperboloid of Two Sheets 	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.

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