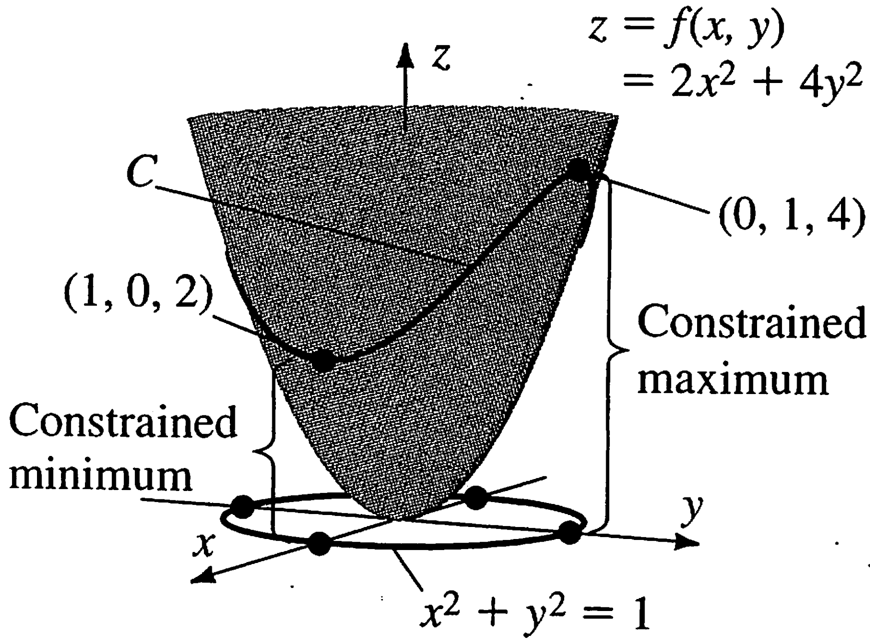
Lagrange Multipliers

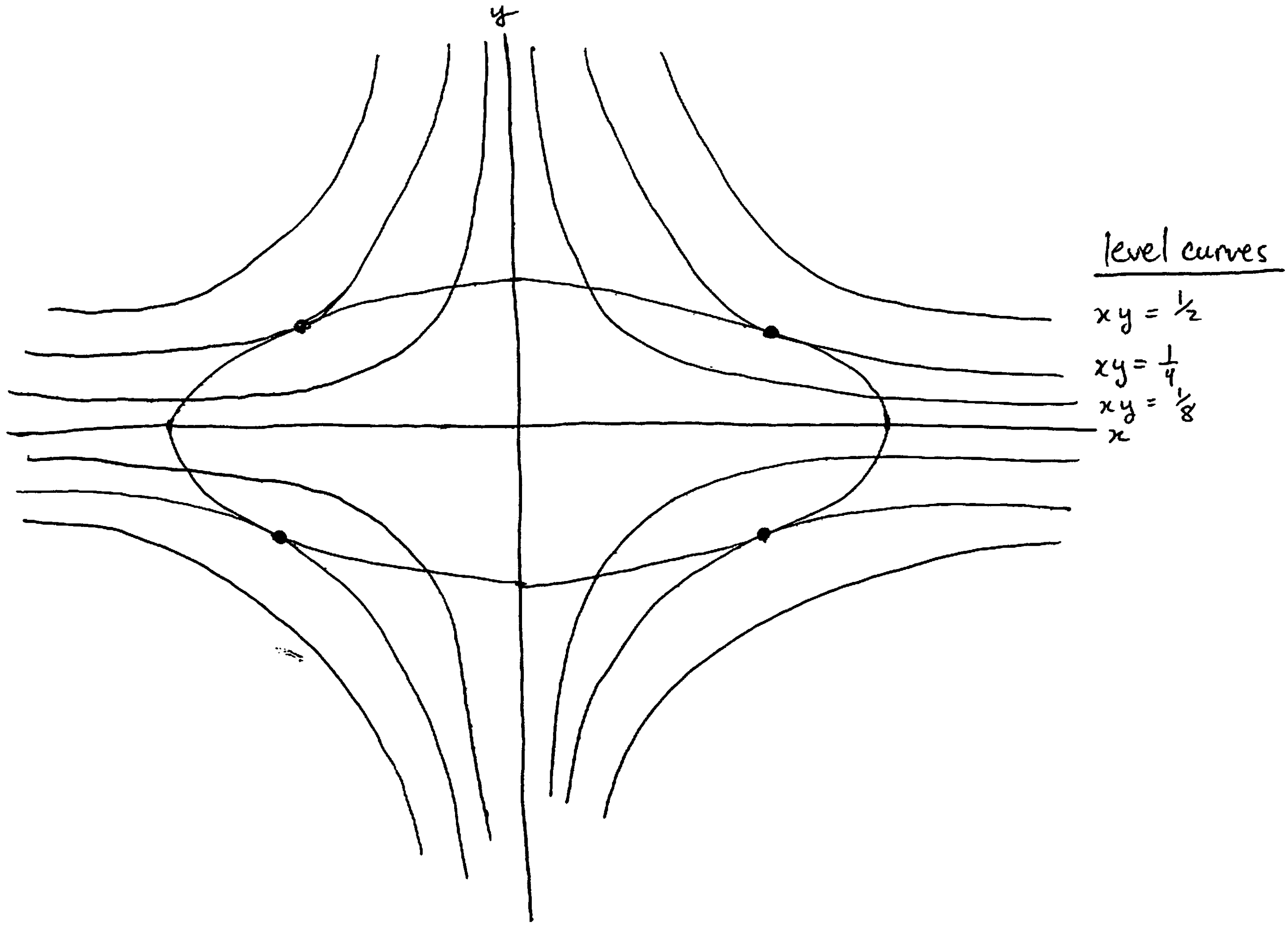
A method for solving optimization problems subject to a constraint, i.e. finding point(s) (x_0, y_0, z_0) at which $g(x_0, y_0, z_0) = k$ and $f(x_0, y_0, z_0)$ is greatest/least among these.



82. Figure 18.8.1 Constrained extrema.
Berkey/Blanchard, Calculus, 3/e

Illustration of the idea in 2 variables

Find min/max of $f(x,y) = xy$ on the ellipse $\underbrace{x^2+4y^2}_{g(x,y)}=1$.



- at extrema (dark dots) ellipse & level curves are <u>tangent</u>, so have same <u>normal direction</u>, i.e. ∇g parallel to ∇f

i.e. $\boxed{\nabla f = \lambda \nabla g}$

λ is the "Lagrange multiplier"

So, we must solve a system of 3 equations:

$$\boxed{\begin{array}{ll} 1) & f_x(x,y) = \lambda g_x(x,y) \\ 2) & f_y(x,y) = \lambda g_y(x,y) \\ 3) & g(x,y) = c \end{array}}$$

($c=1$ in example above)

There are 3 unknowns: , and .

In our example,

1) $y = \lambda(2x)$

2) $x = \lambda(8y)$

3) $x^2 + 4y^2 = 1$

Combine 1) with 2)

Check possible solutions against 3)

We obtain 4 solutions:

x	y	$f(x,y) = xy$	classification

<u>Note</u>: If the question asked for min/max of $f(x,y)=xy$ on the disk $x^2+4y^2 \le 1$, then we would

1) Find critical points of $f(x,y)$ and see if any lie inside $x^2+4y^2 \le 1$.

2) Use the method of Lagrange Multipliers to find the possible max/min points on the boundary.

3) Compare values of $f(x,y)$ at the points found in 1) & 2).

<u>Note</u>: There is no general method for solving the equations obtained by the Lagrange Multipliers method. It is important to do examples to get good at it..