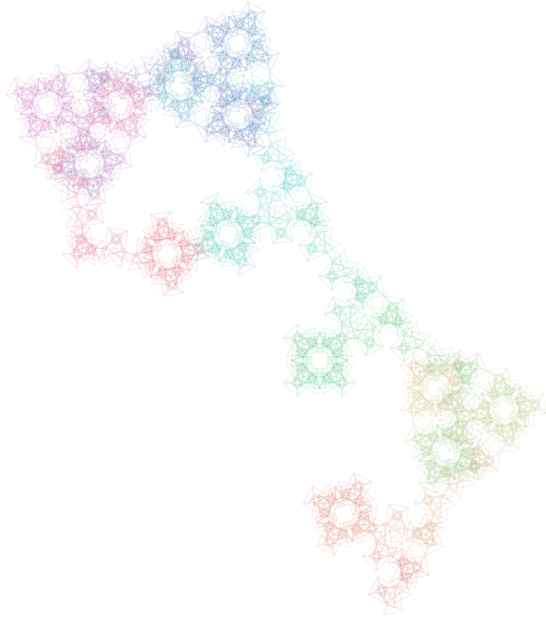


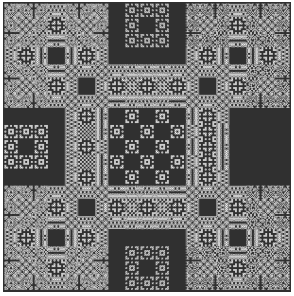
Numberscope

Katherine Stange

University of Colorado, Boulder
Experimental Mathematics Lab
JMM, January 18, 2020

Follow along at
math.katestange.net/illustration/numberscope

Experimental Mathematics Lab at CU Boulder



Part of a growing movement of Geometry Labs United.

Outreach, experimentation, computation, visualization, pedagogy, research.

Numberscope Contributors

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On-Line Encyclopedia of Integer Sequences

The OEIS Foundation is supported by donations from users of the OEIS and by a grant from the Simons Foundation.

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The On-Line Encyclopedia of Integer Sequences® (OEIS®)

Enter a sequence, word, or sequence number:

1,2,3,6,11,23,47,106,235

Search Hints Welcome Video

For more information about the Encyclopedia, see the Welcome page.

Languages: English Shqip العربية Bangla Български Català 中文 (正體字, 简化字 (1), 简化字 (2))
Hrvatski Čeština Dansk Nederlands Esperanto Eesti فارسی Suomi Français Deutsch Ελληνικά ગુજરાતી עברית
हिंदी Magyar Igbo Bahasa Indonesia Italiano 日本語 ಕನ್ನಡ 한국어 Lietuvių मराठी Bokmål Nynorsk Polski Português
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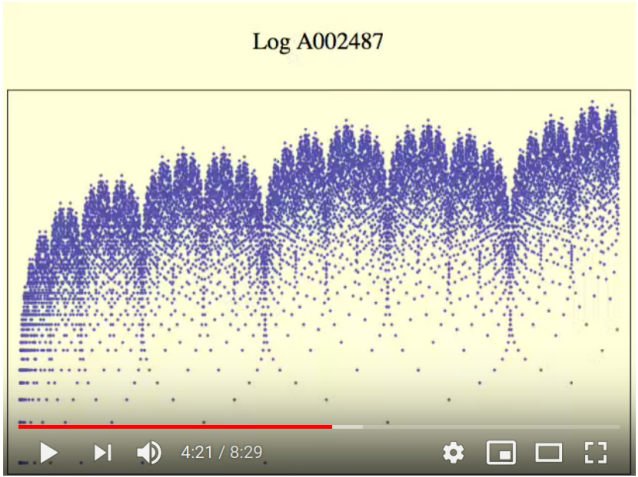
On-Line Encyclopedia of Integer Sequences: The Movie



The OEIS Movie

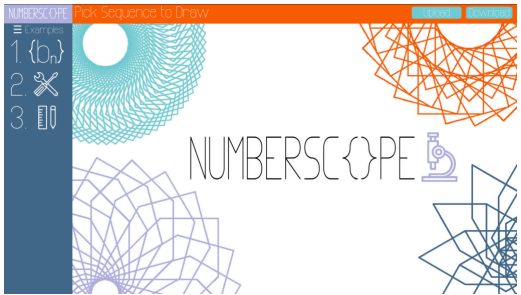
Numberscope: the dream

An online tool that easily pairs a sequence (e.g. input OEIS number) with a visualization tool (e.g. graph).

Audience: researchers, citizen scientists, artists, anyone.

Community extensible: open source, community wiki, API for creating and contributing visualization methods, sequence input etc.

What might we visualize, though?



growth rate

divisibility properties

self-similarity

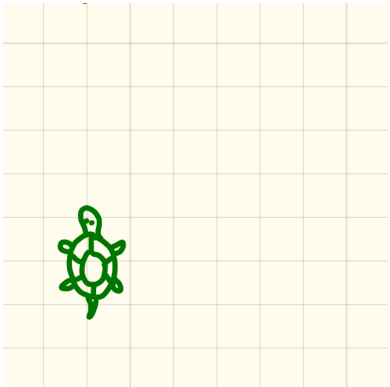
fractal nature

substring statistics

modular periodicity

…

Turtle on a Sequence

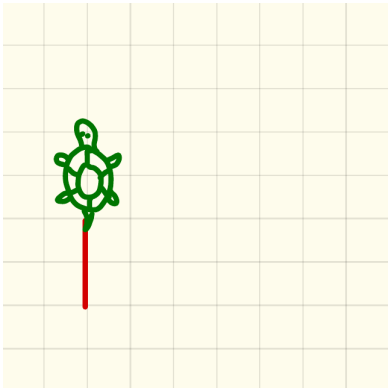


Sequence: 0, 1, 0, 0, …

0: 90 degrees, 1 step

1: 270 degrees, 2 steps

Turtle on a Sequence

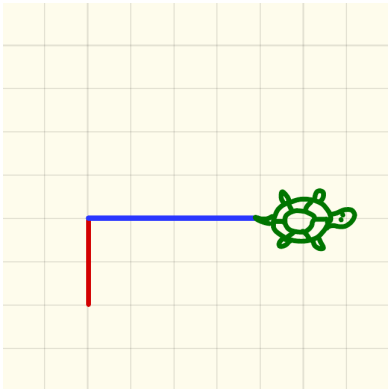


Sequence: 0, 1, 0, 0, …

0: 90 degrees, 1 step

1: 270 degrees, 2 steps

Turtle on a Sequence

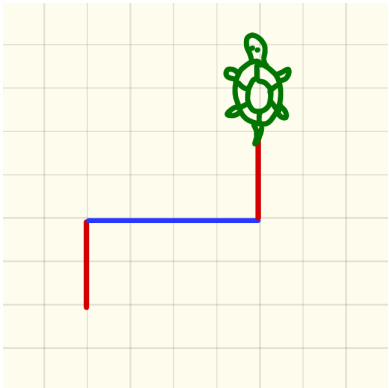


Sequence: 0, 1, 0, 0, …

0: 90 degrees, 1 step

1: 270 degrees, 2 steps

Turtle on a Sequence

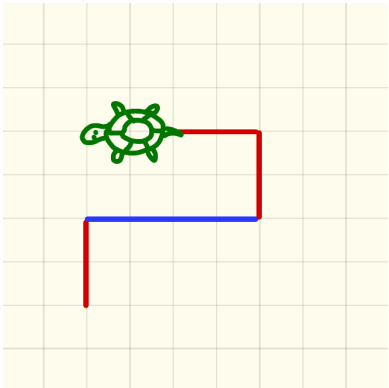


Sequence: 0, 1, 0, 0, …

0: 90 degrees, 1 step

1: 270 degrees, 2 steps

Turtle on a Sequence



Sequence: 0, 1, 0, 0, …

0: 90 degrees, 1 step

1: 270 degrees, 2 steps

Turtle on a Sequence

Hofstadter Figure-Figure A005228

3,7,12,18,26,35,45,56,69,83,98,114,...

press h for help

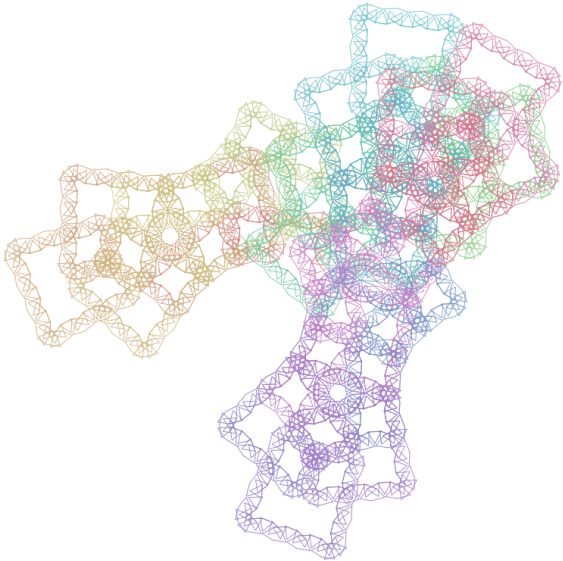
9960 length (f/g; v/b)
3 modulus (m/n; o/p)

Turtle Rules:

press a/x to add/remove

Term	Angle	Speed	Steps	Speed
0	137.0	0.000	3.0	0.00
1	0.0	0.000	2.0	0.00
2	105.0	0.000	1.0	0.00

Turtle on a Sequence



2-adic val of Z

0,0,1,0,2,0,1,0,3,0,1,0,2,0,1,0,4,0,...

press h for help

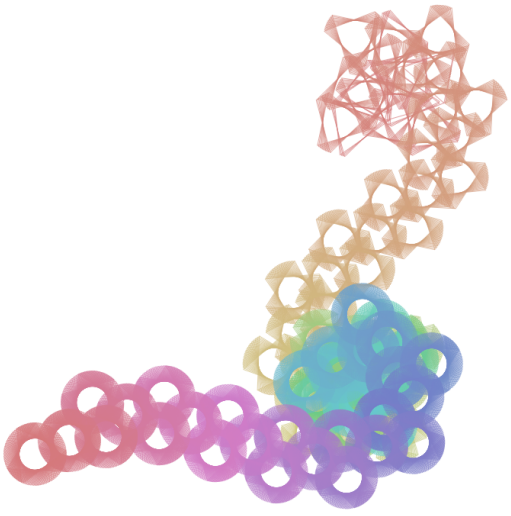
9960 length (f/g; v/b)
2 modulus (m/n; o/p)

Turtle Rules:

press a/x to add/remove

Term	Angle	Speed	Steps	Speed
0	120.0	0.000	8.0	0.00
1	24.0	0.000	7.0	0.00

Turtle on a Sequence



Number of divisors of n
A000005

2,2,3,2,4,2,4,3,4,2,6,2,4,4,5,2,6,2,...

press h for help

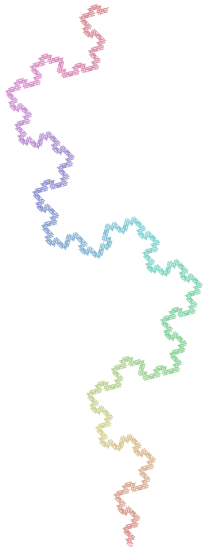
9960 length (f/g; v/b)
2 modulus (m/n; o/p)

Turtle Rules:

press a/x to add/remove

Term	Angle	Speed	Steps	Speed
0	119.0	0.000	8.0	0.00
1	60.0	0.000	9.0	0.00

Turtle on a Sequence



Thue-Morse
A010060

1,1,0,1,0,0,1,1,0,0,1,0,1,1,0,1,0,0,...

press h for help

9960	length (f/g; v/b)
2	modulus (m/n; o/p)

Turtle Rules:

press a/x to add/remove

Term	Angle	Speed	Steps	Speed
0	0.0	0.000	1.0	0.00
1	60.0	0.000	0.0	0.00

Turtle on a Sequence

Continued fraction Pi
A001203

7,15,1,292,1,1,1,2,1,3,1,14,2,1,1,2,...

press h for help

9960 length (f/g; v/b)
0 modulus (m/n; o/p)

Turtle Rules:

press a/x to add/remove

Term	Angle	Speed	Steps	Speed
0	0.0	0.000	0.0	0.00
1	0.0	0.000	1.0	0.00
2	1.0	0.000	0.0	0.00
3	2.0	0.000	0.0	0.00
4	4.0	0.000	0.0	0.00
5	8.0	0.000	0.0	0.00
6	16.0	0.000	0.0	0.00
7	32.0	0.000	0.0	0.00
8	64.0	0.000	0.0	0.00
9	128.0	0.000	0.0	0.00

Self-Similarity Telescope

	a_1	a_2	a_3	a_4	a_5
a_n	a_1	a_2	a_3	a_4	a_5
a_{n+1}	a_2	a_3	a_4	a_5	a_6
a_{n+2}	a_3	a_4	a_5	a_6	a_7
a_{n+3}	a_4	a_5	a_6	a_7	a_8
a_{n+4}	a_5	a_6	a_7	a_8	a_9

contraction $= 1$

translation $= 1$

Self-Similarity Telescope

	a_1	a_2	a_3	a_4	a_5
a_{2n}	a_2	a_4	a_6	a_8	a_{10}
a_{2n+3}	a_5	a_7	a_9	a_{11}	a_{13}
a_{2n+6}	a_8	a_{10}	a_{12}	a_{14}	a_{16}
a_{2n+9}	a_{11}	a_{13}	a_{15}	a_{17}	a_{19}
a_{2n+12}	a_{14}	a_{16}	a_{18}	a_{20}	a_{22}

contraction $= 2$

translation $= 3$

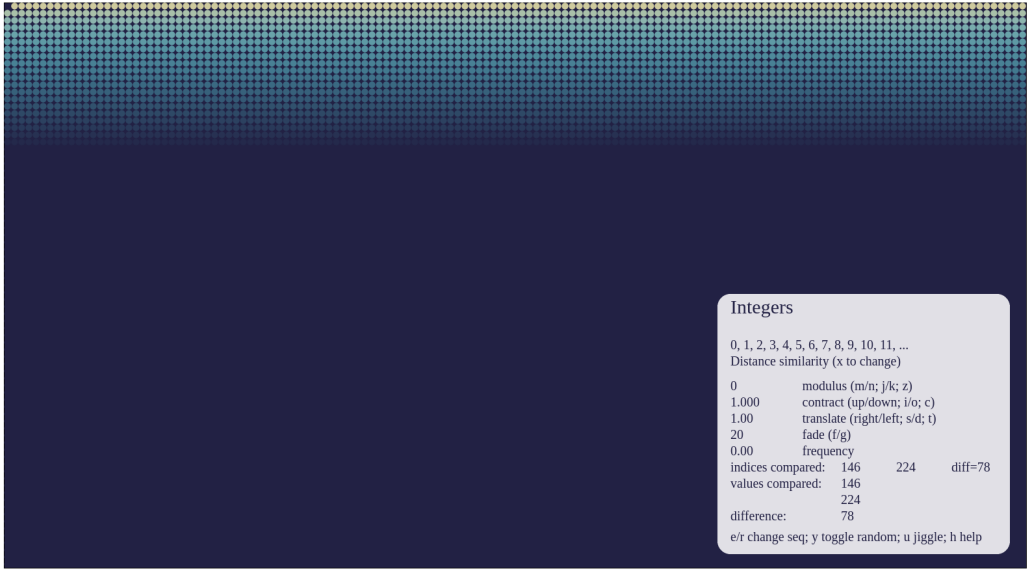
Self-Similarity Telescope

	a_1	a_2	a_3	a_4	a_5
a_n	a_1	a_2	a_3	a_4	a_5
a_{n+1}	a_2	a_3	a_4	a_5	a_6
a_{n+2}	a_3	a_4	a_5	a_6	a_7
a_{n+3}	a_4	a_5	a_6	a_7	a_8
a_{n+4}	a_5	a_6	a_7	a_8	a_9

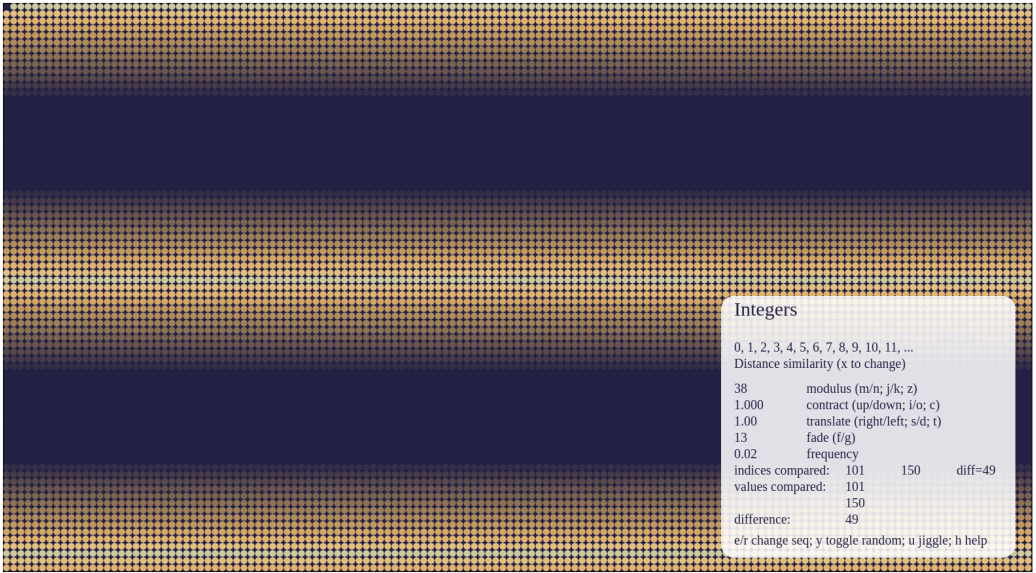
Compare the highlighted term with the column header.
Colour according to (3 modes):

- Distance similarity: $|a_i - a_j|$
- Divisibility detection: $\gcd(a_i, a_j)$
- p-adic similarity: $|v_p(a_i) - v_p(a_j)|$

Self-Similarity Telescope

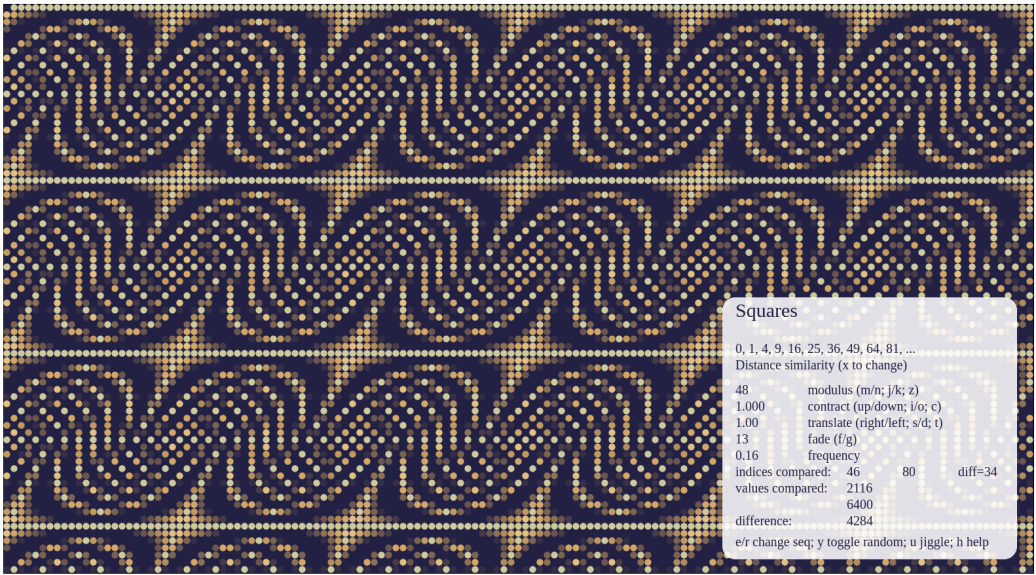


Self-Similarity Telescope

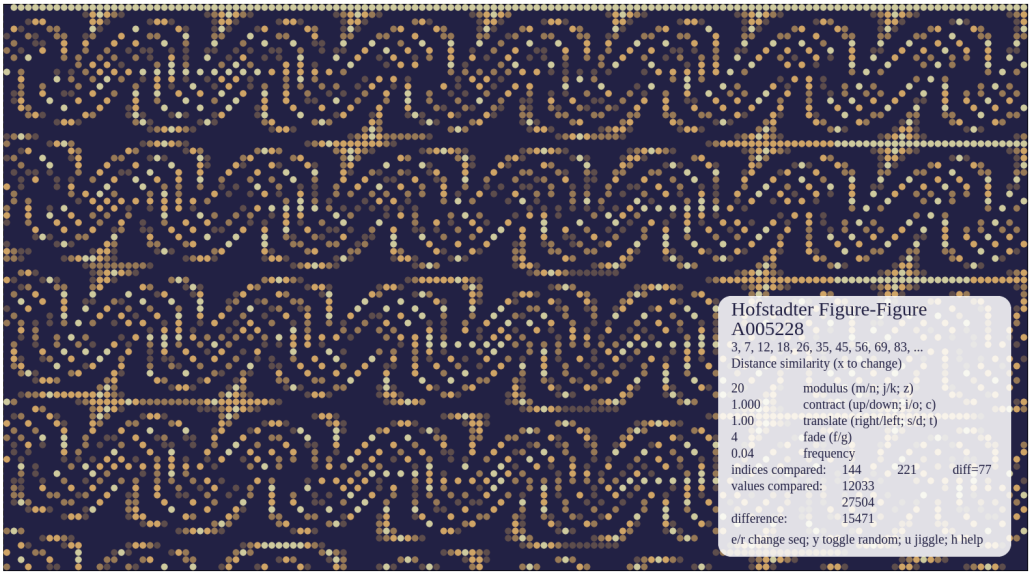


Self-Similarity Telescope

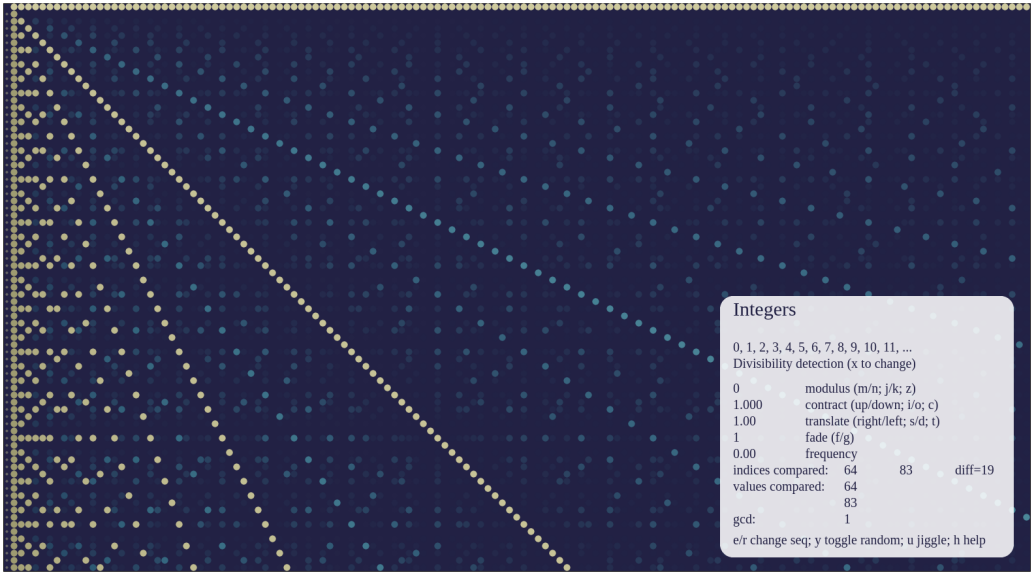
Self-Similarity Telescope



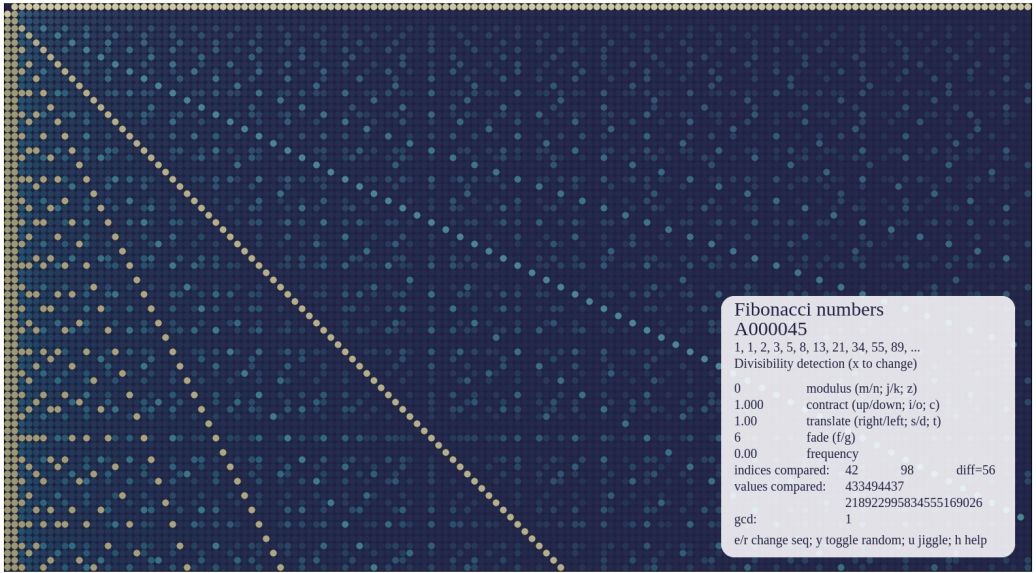
Self-Similarity Telescope



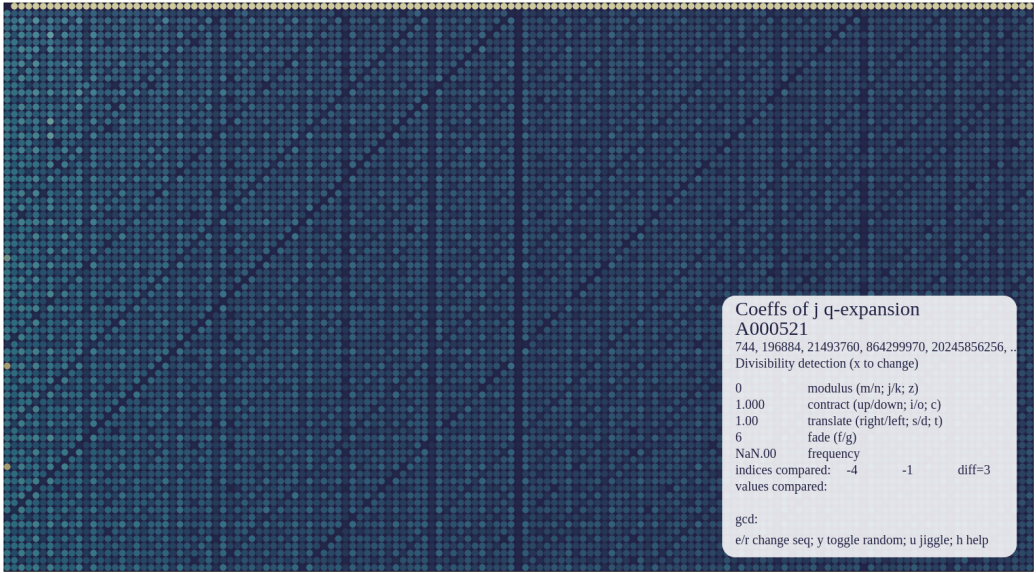
Self-Similarity Telescope



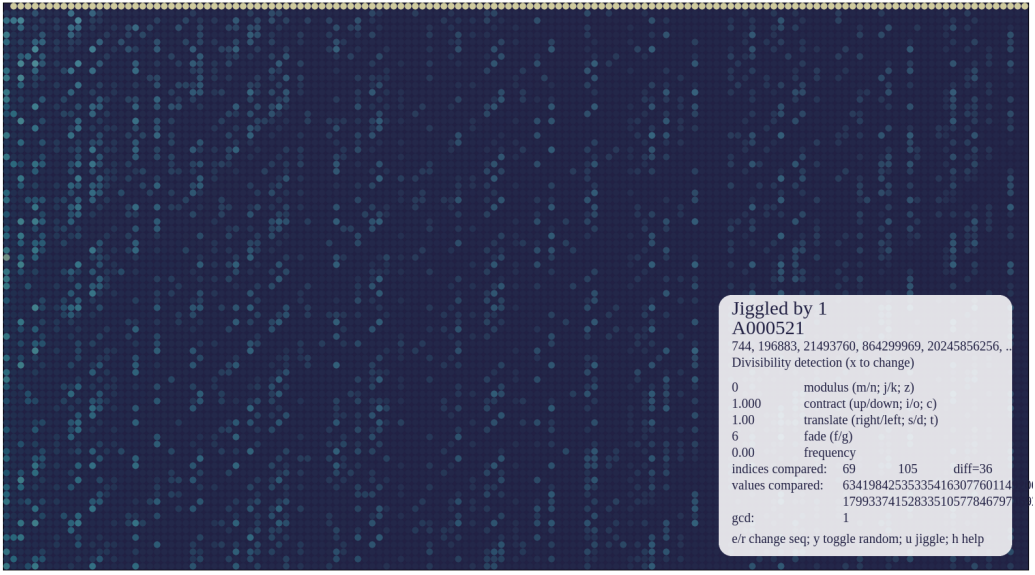
Self-Similarity Telescope



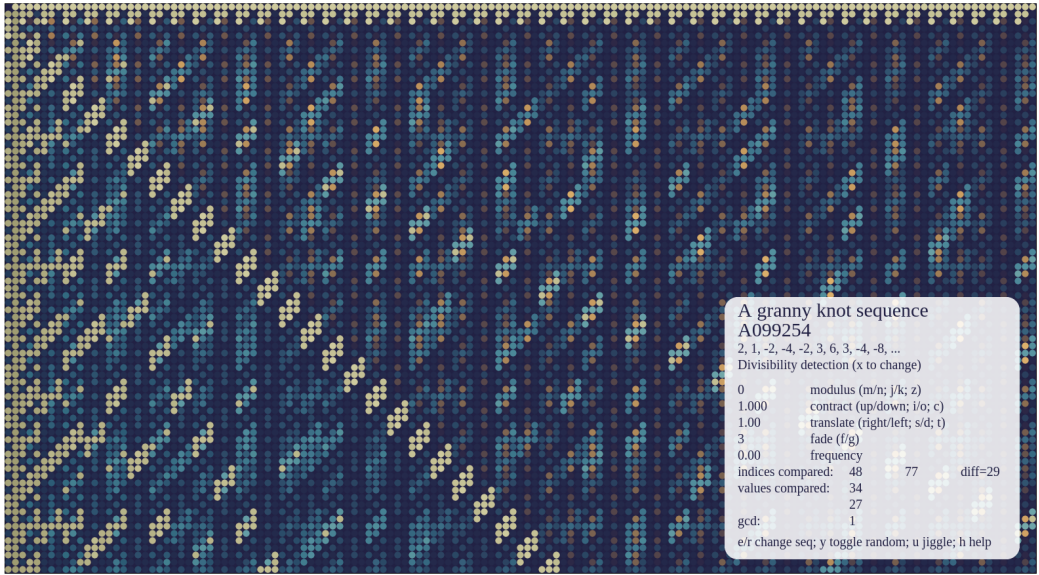
Self-Similarity Telescope



Self-Similarity Telescope



Self-Similarity Telescope



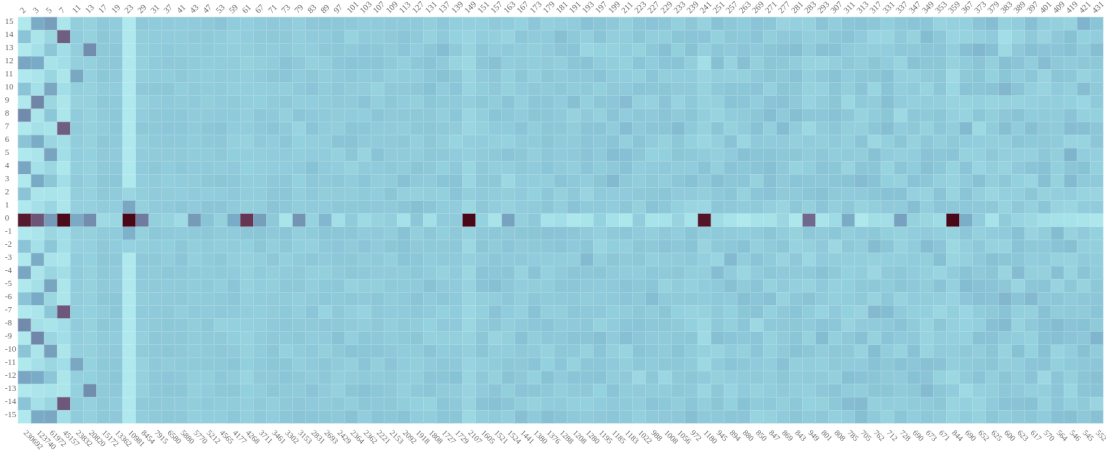
Prime Filter

	2	3	5	7	11
$a_n + 2$	0	0	0	0	0
$a_n + 1$	0	0	0	0	0
a_n	0	0	0	0	0
$a_n - 1$	0	0	0	0	0
$a_n - 2$	0	0	0	0	0

At each coordinate (prime, sequence), we record with a darker colour if the first N terms are frequently divisible by the prime.

Precisely, a histogram of the *sum of the valuations mod p*, or *the frequency of 0 mod p* (two modes).

Prime Filter



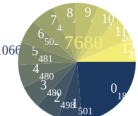
Ramanujan Tau

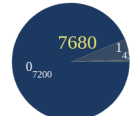
-24, 252, -1472, 4830, -6048, -16744, 84480, -113643, -115920, 534612, -370944, -577738, 401856, 1217160, 987136, -6905934, 2727432, 1066

Valuations, total sum

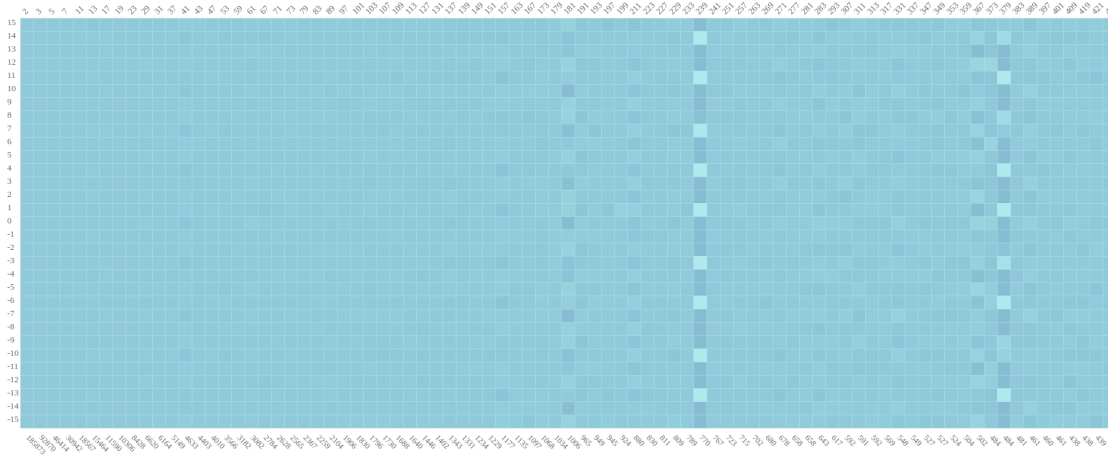
Terms beginning at 0, ending at 8000

Press h for help





Prime Filter



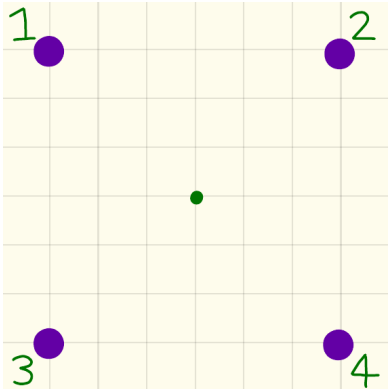
Beatty (floor n*(sqrt2))

1, 2, 4, 5, 7, 8, 9, 11, 12, 14, 15, 16, 18, 19, 21, 22, 24, 25, 26, 28, 29, 31, 32, 33, 35, 36, 38, 39, 41, 42, 43, 45, 46, 48, 49, 50, ...

Valuations, total sum

Terms beginning at 0, ending at 10000

Chaos Game

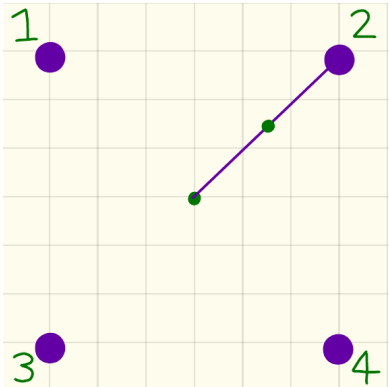


Sequence: 2, 4, 3, 3, …

Start at origin…

At each term, step halfway to the corresponding corner.

Chaos Game

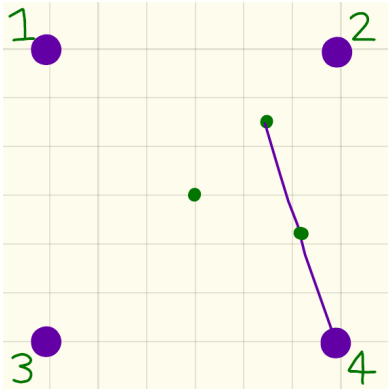


Sequence: 2, 4, 3, 3, …

Start at origin…

At each term, step halfway to the corresponding corner.

Chaos Game

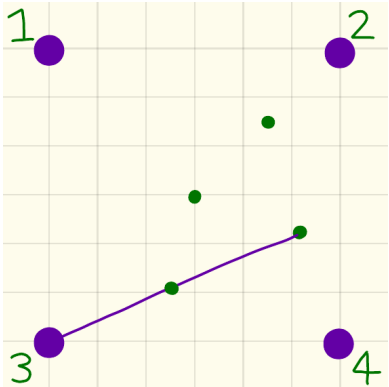


Sequence: 2, 4, 3, 3, …

Start at origin…

At each term, step halfway to the corresponding corner.

Chaos Game

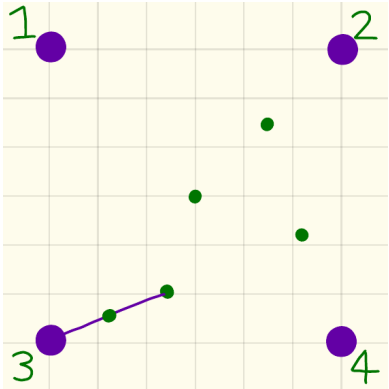


Sequence: 2, 4, 3, 3, …

Start at origin…

At each term, step halfway to the corresponding corner.

Chaos Game

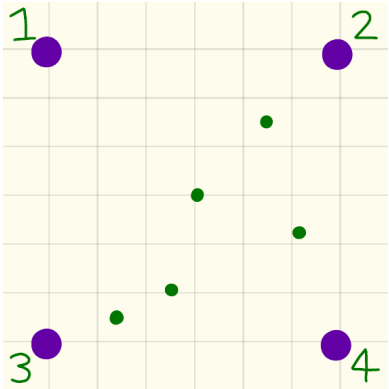


Sequence: 2, 4, 3, 3, …

Start at origin…

At each term, step halfway to the corresponding corner.

Chaos Game

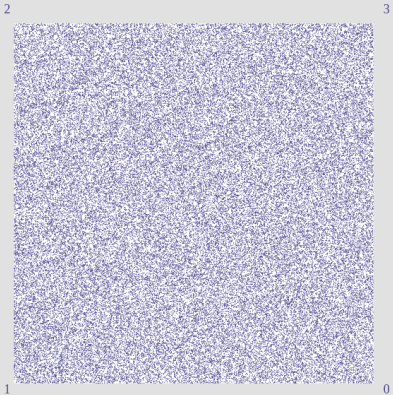


Sequence: 2, 4, 3, 3, …

Start at origin…

At each term, step halfway to the corresponding corner.

Prime Filter



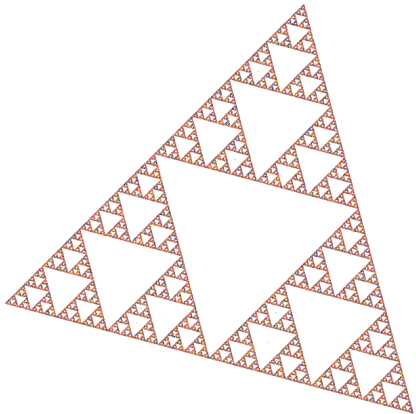
Random Modulo 4

2, 3, 3, 3, 2, 0, 3, 1, 3, 0, 3, 3, 3, 1, 1, ...

Modulus (m/n):	4
Fractional step (u/i):	0.50
Number of Walkers (t/y):	1
Size of dots (f/g):	1
Darkness (j/k):	250
Head fade (v/b):	10
Color style (c):	colour by walker

q: toggle random
l: toggle background
p: change palette

Prime Filter

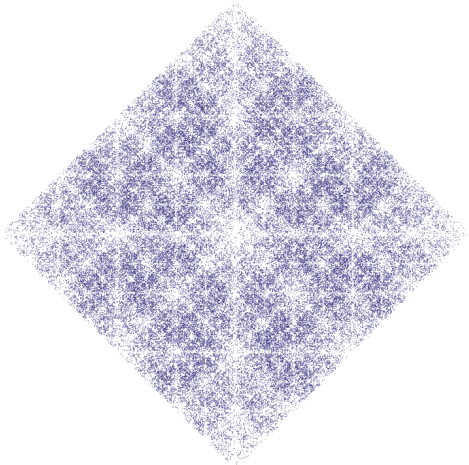


Random Modulo 3
A000005
0, 2, 2, 1, 0, 1, 1, 1, 1, 2, 1, 0, 2, 1, 0, ...

Modulus (m/n):	3
Fractional step (u/i):	0.50
Number of Walkers (t/y):	6
Size of dots (f/g):	1
Darkness (j/k):	250
Head fade (v/b):	10
Color style (c):	colour by walker

q: toggle random
l: toggle background
p: change palette

Prime Filter

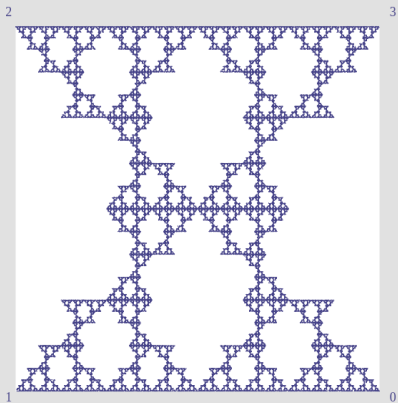


Prime numbers
A000040
3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, ...

Modulus (m/n):	8
Fractional step (u/i):	0.50
Number of Walkers (t/y):	1
Size of dots (f/g):	1
Darkness (j/k):	250
Head fade (v/b):	10
Color style (c):	colour by walker

q: toggle random
l: toggle background
p: change palette

Prime Filter



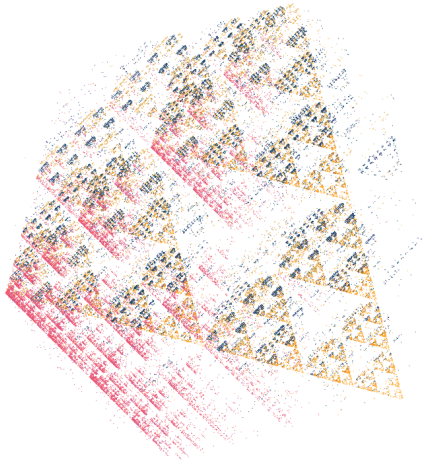
3^n modulo 1000003

1, 3, 9, 27, 81, 243, 729, 2187, 6561, 19683, ...

Modulus (m/n):	4
Fractional step (u/i):	0.50
Number of Walkers (t/y):	1
Size of dots (f/g):	1
Darkness (j/k):	250
Head fade (v/b):	10
Color style (c):	colour by walker

q: toggle random
l: toggle background
p: change palette

Prime Filter



Number of divisors of n
A000005

2, 2, 3, 2, 4, 2, 4, 3, 4, 2, 6, 2, 4, 4, 5, ...

Modulus (m/n):	12
Fractional step (u/i):	0.50
Number of Walkers (t/y):	6
Size of dots (f/g):	1
Darkness (j/k):	250
Head fade (v/b):	10
Color style (c):	colour by walker

q: toggle random
l: toggle background
p: change palette

Thank you!

If you are interested in being a beta tester, please email me.

If you have a favourite integer sequence, please email me.

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