

APPUZZLE

This is a puzzle to reassemble an Apollonian circle packing. The goal is to fit all the circles into the circular puzzle board. The circles have their curvatures marked, and, mathematically speaking, there should be only one solution. However, since the circles pieces have some wiggle from the wood lost in the laser cutting process, the puzzle pieces are actually not exact enough to obtain the correct solution without using the mathematical structure of the Apollonian circle packing itself (for example, curvatures 119 and 123 are, for all practical purposes, the same size).

Here's the rule:

Any four circles which are all mutually tangent are called a *Descartes quadruple*, and their curvatures (marked on the circles) satisfy

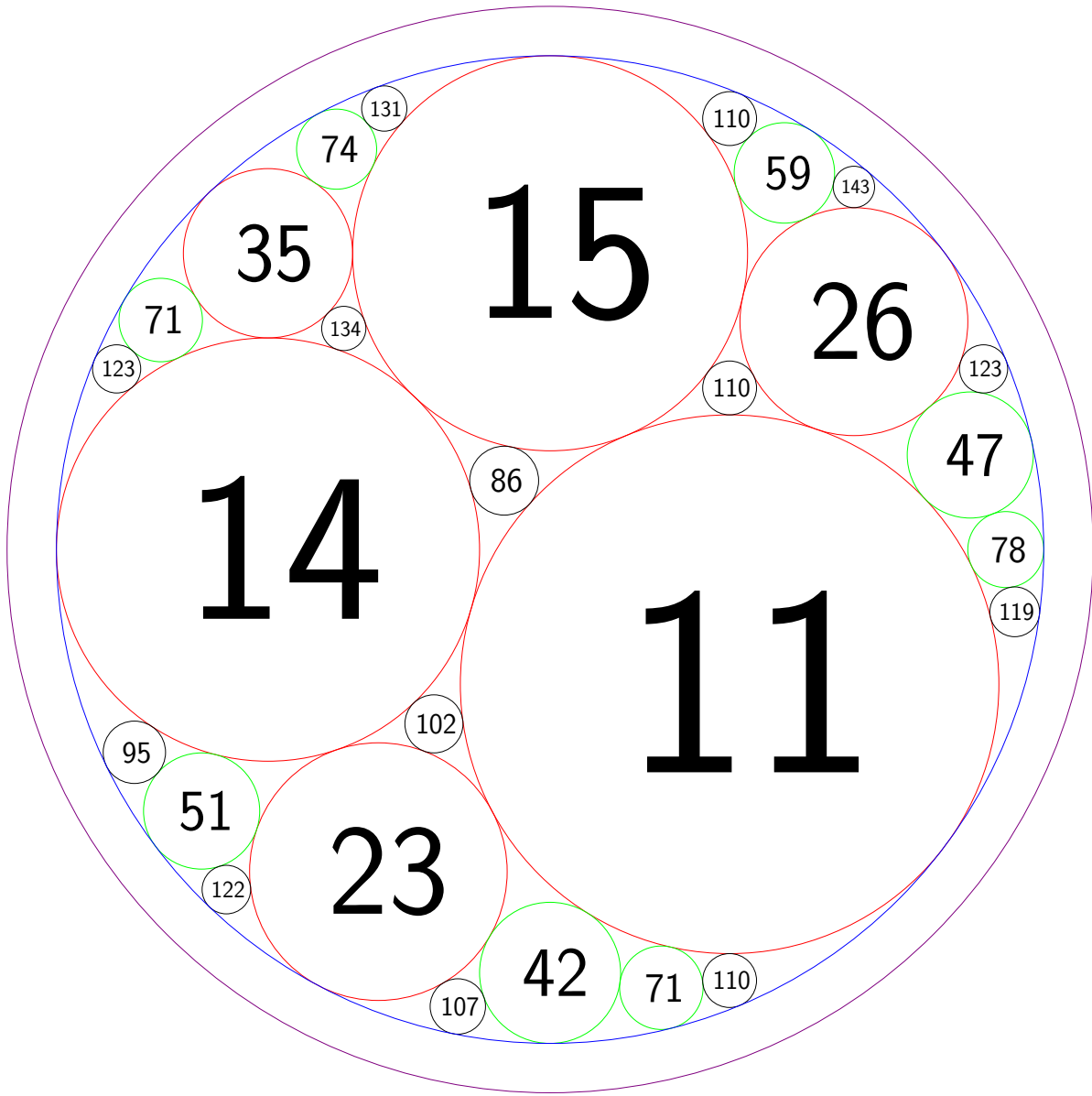
$$2(a^2 + b^2 + c^2 + d^2) = (a + b + c + d)^2.$$

It is a theorem of Apollonius that if one has three mutually tangent circles of curvatures a , b and c , then there are exactly two ways to add a fourth circle to make a Descartes quadruple. From Descartes' quadratic form above, these two new circles have curvatures d_1 and d_2 which satisfy

$$d_1 + d_2 = 2(a + b + c).$$

The solution is on the back of this page.

— Daniel Martin and Katherine Stange



Instructions for creating your own Apuzzle on a Glowforge:

- (1) Upload the pdf file of the puzzle.
- (2) On one piece of wood, score everything except the outer circle, which you should cut (last). This creates the backboard. Note: if you wish to leave the solution showing on the bottom of the puzzle (viewed from below), you may wish to use the reflected PDF (it depends what effect you want).
- (3) On a second piece of wood, score the numbers and then cut the circles in order from small to big.
- (4) Throw away the little triangles you are left with.
- (5) Glue (with wood glue) the resulting “outer ring” onto the backboard, to create the puzzle working area. (You can choose whether to leave the solution showing on the bottom of the puzzle or on the inside.) Leave it under a heavy book for some hours so the glue can dry.