

# KATHERINE E. STANGE

UNIVERSITY OF COLORADO, BOULDER  
[math.colorado.edu/~kstange](http://math.colorado.edu/~kstange)

## RESEARCH AREAS

Algebraic number theory and arithmetic geometry, including Kleinian groups, elliptic curves and abelian varieties, integer sequences, cryptography.

## EDUCATION

*Ph.D.* 2008      Brown University  
under Joseph H. SILVERMAN

*M.Sc.* 2003      Brown University

*B.Math.* 2001    University of Waterloo

## ACADEMIC POSITIONS

<i>Faculty</i>	2018-present	The University of Colorado, Boulder Associate Professor
	2012-2018	The University of Colorado, Boulder Assistant Professor
<i>Visiting</i>	Fall 2019	The Institute for Computational and Experimental Research in Mathematics (ICERM) Research Fellow, Semester program on Illustrating Mathematics
<i>Postdoctoral</i>	2011-2012	Stanford University NSF Postdoctoral Fellow
	2009-2011	Simon Fraser University, Pacific Institute for the Mathematical Sciences, and the University of British Columbia NSERC/PIMS/NSF Postdoctoral Fellow
	2008-2009	Harvard University NSF Postdoctoral Fellow and Junior Lecturer

## RESEARCH AWARDS

<i>Research Grants</i>	2022-2023	CU Boulder Growth Grant, \$8,721.56
	2021-2022	Simons Fellows, Simons Foundation, \$94,489
	2018-2019	Co-PI, CU Boulder RIO QuEST, \$50,000
	2017-2024	PI, NSF CAREER, CNS-1652238, \$539,975
	2016-2018	PI, NSF EAGER, DMS-1643552, \$200,000
	2016-2017	PI, NSA, Young Investigators, \$40,000
<i>Conference Grants</i>	2014-2015	PI, NSA, Young Investigators, \$40,000
	2020	Co-PI, NSA, \$15,000 (CTNA XVI)
	2019-2022	Co-PI, NSF, DMS 1936672, \$12,735 (FRNTD)
<i>Postdoctoral Awards</i>	2019	Co-PI, CU Boulder RIO \$1,625 (FRNTD)
	2008-2012	NSF MSPRF, \$108K
	2009-2011	NSERC (Canada) Postdoctoral Fellowship, \$80K "Most outstanding candidate at the Postdoctoral level, Mathematics"
	2009-2011	PIMS Postdoctoral Fellowship

PUBLICATIONS

- Mathematical Cryptology* (Mathcrypt 2023) **Factoring using multiplicative relations modulo  $n$ : a subexponential algorithm inspired by the index calculus**  
Katherine E. STANGE  
*Mathematical Cryptology*, 3(2) (2023), 2-10. (Mathcrypt 2023 special issue)
- La Matematica* **Orienteering with one endomorphism**  
Sarah ARPIN, Mingjie CHEN, Kristin E. LAUTER, Renate SCHEIDLER, Katherine E. STANGE and Ha T. N. TRAN  
*La Matematica* (2023) *early online publication*
- Experimental Mathematics* **Algebraic Number Starscapes**  
Edmund HARRISS, Katherine E. STANGE, Steve TRETTEL  
*Experimental Mathematics*, 31:4 (2022) 1098–1149.
- Involve* **Monogenic fields arising from trinomials**  
Ryan IBARRA, Henry LEMBECK, Mohammad OZASLAN, Hanson SMITH, Katherine E. STANGE  
*Experimental Mathematics*, 15:2 (2022) 299–317.
- CRYPTO 2021 **Improved torsion point attacks on SIDH variants**  
Victoria DE QUEHEN, Péter KUTAS, Chris LEONARDI, Chloe MARTINDALE, Lorenz PANNY, Christophe PETIT, Katherine E. STANGE  
*Advances in Cryptology – CRYPTO 2021*, Part 3, vol. 12827 of *Springer Lecture Notes in Computer Science* (2021), 432–470.
- SIAM Journal on Applied Algebra and Geometry* **Algebraic aspects of solving Ring-LWE, including ring-based improvements in the Blum-Kalai-Wasserman algorithm**  
Katherine E. STANGE  
*SIAM Journal on Applied Algebra and Geometry*, 5:2 (2021), 366–387.
- Journal of Number Theory* **A family of monogenic  $S_4$  quartic fields arising from elliptic curves**  
T. Alden GASSERT, Hanson SMITH and Katherine E. STANGE  
*Journal of Number Theory*, 197 (2019), pp. 361-382.
- Compositio Mathematica* **Local-Global Principles in Circle Packings**  
Elena FUCHS, Katherine E. STANGE, and Xin ZHANG  
*Compositio Mathematica*, 155:6 (2019), pp. 1118-1170.
- SIAM Journal of Applied Algebra and Geometry* **Attacks on the Search-RLWE problem with small errors**  
Hao CHEN, Kristin LAUTER and Katherine E. STANGE  
*SIAM Journal of Applied Algebra and Geometry*, 1-1 (2019), pp. 665–682.
- Transactions of the AMS* **The dynamics of super-Apollonian continued fractions**  
Sneha CHAUBEY, Elena FUCHS, Robert HINES and Katherine E. STANGE  
*Transactions of the American Mathematical Society*, 372 (2019), pp. 2287–2334.
- Transactions of the AMS* **The Apollonian structure of Bianchi groups**  
Katherine E. STANGE  
*Transactions of the American Mathematical Society*, 370 (2018), pp. 6169–6219.
- SAC 2016 **Security Considerations for Galois Non-dual RLWE Families**  
Hao CHEN, Kristin LAUTER and Katherine E. STANGE  
*Selected Areas in Cryptography 2016 – SAC 2016*, LNCS vol 10532, pp. 443–462.
- International Mathematics Research Notices* **Visualising the arithmetic of imaginary quadratic fields**  
Katherine E. STANGE  
*International Mathematics Research Notices*, 12 (2018), pp. 3908–3938.
- New York Journal of Mathematics* **Index divisibility in dynamical sequences and cyclic orbits modulo  $p$**   
Annie S. CHEN, T. Alden GASSERT and Katherine E. STANGE  
*New York Journal of Mathematics*, 2017.23, pp. 1045–1063.
- International Mathematics Research Notices* **Arithmetic properties of the Frobenius traces defined by a rational abelian variety**  
Alina COJOCARU, Rachel DAVIS and Alice SILVERBERG and Katherine E. STANGE with two appendices by J-P. SERRE  
*International Mathematics Research Notices*, 2017.12, pp. 3557–3602.
- Expositiones Mathematicae* **The sensual Apollonian circle packing**

- Katherine E. STANGE  
*Expositiones Mathematicae*, 34.4, pp. 364-395.
- Research Directions in Number Theory*  
**RLWE Cryptography for the Number Theorist**  
Yara ELIAS, Kristin E. LAUTER, Ekin OZMAN and Katherine E. STANGE  
*Research Directions in Number Theory: Proceedings of the 2014 WIN<sub>3</sub> Workshop*, vol. 3 of *Association for Women in Mathematics Series*, pp. 271–290.
- Canadian Journal of Mathematics*  
**Integral points on elliptic curves and explicit valuations of division polynomials**  
Katherine E. STANGE  
*Canadian Journal of Mathematics*, 68.5, pp. 1120–1158.
- CRYPTO 2015  
**Weak instances of Ring-LWE**  
Yara ELIAS, Kristin E. LAUTER, Ekin OZMAN and Katherine E. STANGE  
*Advances in Cryptology – CRYPTO 2015*, Part I, vol. 9215 of *Springer Lecture Notes in Computer Science*, pp. 63–92.
- Proceedings of the AMS*  
**A duality principle for selection games**  
Lionel LEVINE, Scott SHEFFIELD and Katherine E. STANGE  
*Proceedings of the American Mathematical Society*, 141, pp. 4349-4356.
- American Mathematical Monthly*  
**How to make the most of a shared meal: plan the last bite first**  
Lionel LEVINE and Katherine E. STANGE  
*American Mathematical Monthly*, 119.7, pp. 550-565.
- Journal of the Australian Mathematical Society*  
**Algebraic divisibility sequences over function fields**  
Patrick INGRAM, Valéry MAHÉ, Joseph H. SILVERMAN, Katherine E. STANGE and Marco STRENG  
*Journal of the Australian Mathematical Society* (special issue dedicated to Alf van der Poorten) 92.1, pp. 99-126.
- Canadian Mathematical Bulletin*  
**Character sums with division polynomials**  
Igor E. SHPARLINSKI and Katherine E. STANGE  
*Canadian Mathematical Bulletin*, 55, pp. 850-857.
- Algebra & Number Theory*  
**Elliptic nets and elliptic curves**  
Katherine E. STANGE  
*Algebra & Number Theory* 5.2, pp. 197-229.
- Experimental Mathematics*  
**Amicable pairs and aliquot cycles for elliptic curves (2011)**  
Joseph H. SILVERMAN and Katherine E. STANGE  
*Experimental Mathematics* 20.3, pp. 329-357.
- Acta Arithmetica*  
**Terms in elliptic divisibility sequences divisible by their indices**  
Joseph H. SILVERMAN and Katherine E. STANGE  
*Acta Arithmetica* 146.4, pp. 355-378.
- Women in Numbers*  
**Pairings on hyperelliptic curves**  
Jennifer BALAKRISHNAN, Juliana BELDING, Sarah CHISHOLM, Kirsten EISENTRÄGER, Katherine E. STANGE and Edlyn TESKE  
*WIN – Women in Numbers: Research Directions in Number Theory*, Fields Institute Communications 60, pp. 87-120.
- SAC 2008  
**The elliptic curve discrete logarithm problem and equivalent hard problems for elliptic divisibility sequences**  
Kristin LAUTER and Katherine E. STANGE  
*Selected Areas in Cryptography 2008*, vol. 5381 of *Springer Lecture Notes in Computer Science*, pp. 309-327.
- PAIRING 2007  
**The Tate pairing via elliptic nets**  
Katherine E. STANGE  
*Pairing-Based Cryptography – PAIRING 2007*, vol. 4575 of *Springer Lecture Notes in Computer Science*, pp. 329-348.

#### TO APPEAR

- The Computer Journal (CFAIL 2022)*  
**Failing to hash into supersingular graphs**  
Jeremy BOOHER, Ross BOWDEN, Javad DOLISKANI, Tako Boris FOUOTSA, Steven D. GALBRAITH, Sabrina KUNZWEILER, Simon-Philipp MERZ, Christophe PETIT, Benjamin SMITH, Katherine E. STANGE, Yan Bo TI, Christelle VINCENT, José Felipe VOLOCH, Charlotte WEITKÄMPER, Lukas ZOBERNIG

Proceedings of  
Women in  
Numbers 5

**Orientations and cycles in supersingular isogeny graphs**

Sarah ARPIN, Mingjie CHEN, Kristin E. LAUTER, Renate SCHEIDLER, Katherine E. STANGE, Ha T. N. TRAN

SCHOLARSHIP OF TEACHING AND LEARNING

PRIMUS

**Standards Based Grading in an Introduction to Abstract Mathematics**

Katherine E. STANGE

PRIMUS: Problems, Resources and Issues in Mathematics Undergraduate Studies 28.9, pp. 797–820.

HONORS

Association for  
Women in  
Mathematics  
Fellow

Class of 2021, “For leadership in the Women in Numbers Network by creating its website (the first of its kind), mentoring early-career researchers, organizing conferences, editing its proceedings volumes, and chairing its steering committee; and for service on AWM committees, including support of other research networks.”

Writing Award

2013 Paul R. Halmos - Lester R. Ford Award for outstanding paper in *The American Mathematical Monthly*, awarded for joint paper with Lionel LEVINE, *How to make the most of a shared meal: plan the last bite first*

Exposition Award

2023 Winner (one of five), Summer of Mathematics Exposition 3  
<https://some.3b1b.co/>, sponsored by 3blue1brown, for YouTube video *Rethinking the real line*.

OTHER ACTIVITIES

Selected  
Expositional  
Writing

*On the importance of illustration for mathematical research*, with Rémi COULON, Gabriel DORFSMAN-HOPKINS, Edmund HARRISS, Martin SKRODZKI, and Glen WHITNEY; *Notices of the American Mathematical Society* to appear January 2024, [arXiv:2307.04636](https://arxiv.org/abs/2307.04636).  
*The Ingenious Physical Factoring Devices of D.N. Lehmer*, *Math Horizons* 30:2 (2022), 8–11., [doi:10.1080/10724117.2022.2112892](https://doi.org/10.1080/10724117.2022.2112892).  
*An illustration in number theory (2019 Lecture Sampler)*, *Notices of the American Mathematical Society* 66:03 (2019), 411–413,  
<https://www.ams.org/journals/notices/201903/rnoti-p411.pdf>.

Pedagogy

*University of British Columbia Postdoctoral Teaching Award*, 2011  
*Brown University Mathematics Outstanding Teaching Award*, 2008  
*Proof of Concept*, Educational YouTube Channel (6K+ subscribers, 230K+ views)  
*TRESTLE Scholar*, CU Boulder, Spring 2017  
*Faculty Teaching Excellence Program Summer Institute*, Summer 2014  
*Selected professional development* Be The Change: Practicing Inclusive Excellence in the Classroom (2019), Graduating Advising Workshop (2017), Inquiry Based Learning Workshop (2016)  
*Sheridan Center Teaching Certificate*, Brown University, 2005  
*Standards Based Grading in a First Proofs Course*, presentation at JMM 2017  
Course materials incl. online videos via University of Colorado Boulder  
ASSETT grant

Courses Taught

*Undergraduate, CU Boulder*: Calculus II, Introduction to Discrete Mathematics (x8), Linear Algebra, Coding and Cryptography (x5), Combinatorics, Introduction to the Theory of Numbers  
*Graduate, CU Boulder*: Introduction to Number Theory (x2), Introduction to Modern Algebra I, Algebraic Number Theory (x4), Topics: Arithmetic in Kleinian Groups, Topics: Elliptic Curves, Topics: Mathematical Cryptography  
*University of British Columbia*: Vector Calculus  
*Harvard University*: Algebraic Number Theory, Mathematics of Symmetry  
*Brown University*: Introductory Calculus, Multivariable Calculus, Linear Algebra

Graduate Schools  
Given

upcoming · Computational Aspects of Thin Groups (Integral packings and number theory), IMS Singapore  
2023/07 · Renormalization and visualization for packing, billiard and surfaces (Number theory through geometry, dynamics and visualization), CIRM Luminy

Selected Talks

2024/03 · Plenary, Southern Regional Number Theory Conference  
2023/08 · Semi-Plenary, The VIth Interdisciplinary International Conference

on Applied Mathematics, Modeling and Computational Science  
 2022/03 · Gathering4Gardner Celebration of Mind  
 2022/12 · Public Lecture, The Pacific Rim Mathematical Association Congress  
 2022/02 · Plenary, Florida Women in Mathematics Day  
 2021/06 · Plenary, Arithmetic Geo., Crypt., and Coding Theory 2021 (CIRM)  
 2020/07 · Lucas Lecturer, The Nineteenth International Conference on  
 Fibonacci Numbers and Their Applications  
 2020/03 · Speaker (with Jordan Ellenberg, moderated by Terry Tao), The  
 National Academies Webinar *Illustrating Mathematics: Abstract Geometry,  
 Concrete Impact*  
 2019/03 · Invited Address, AMS Spring Joint Central and Western Sectional  
 2016/03 · Plenary, Alberta Number Theory Days  
 2016/04 · Plenary, SouthEast Regional Meeting on Numbers  
 2015/09 · Invited, ECC 2015  
 2007/09 · Invited, ECC 2007

*Selected Press*

Featured in *Quanta Magazine*, 2023, work on Apollonian packings  
 Featured in *What's Happening in the Mathematical Sciences*, Volume 12, 2022,  
 work on Apollonian packings and Schmidt arrangements  
 Featured in *New Scientist Magazine*, 2011, work in game theory

*Women in  
 Mathematics*

Co-organizer and refereed proceedings editor, Women in Numbers 3  
 Project Leader at Women in Numbers 4, Women in Numbers 5  
 Chair, Women in Number Theory Steering Committee, 2019-onwards  
 Webmaster, Women in Number Theory Steering Committee, 2016-onwards  
 Member, Women in Number Theory Steering Committee, 2014-onwards  
 AWM Advance NSF Grant, AWM Research Networks Committee, 2017-2021  
 Mentor, AWM Mentor Network, 2016-onwards  
 Invited mentor, AWM Graduate Student Poster Session, JMM 2016, 2017

*Software and  
 Visualization*

Steering Committee, *Illustrating Mathematics*  
 Co-Organizer, *Illustrating Mathematics*, Special Semester Fall 2019, ICERM  
 Director of Development, Numberscope  
 Contributor to Sage Mathematics Software  
 Group leader at Sage Days 33: Women in Sage

*Selected Other  
 Service*

Editorial Board, *Advances in Mathematics of Communications*, 2023 onwards  
 Editorial Board, *Math Horizons*, 2020 onwards  
 Program Co-Chair, MathCrypt 2022  
 Program Committee, ANTS 2020, 2022; MathCrypt 2018, 2021  
 Advisory Boards, Scientific and DEI, Banff IRS, 2022-2023  
 Prize Committee, David P. Robbins Prize Selection Committee, AMS, 2024-2027  
 Microsoft Research Prize Committee, AWM, 2024-2028

*Early Research  
 Experiences*

Director, CU Experimental Mathematics Lab, 2017-onwards Mathematics Lab  
 Project Leader, 2017-onwards  
 Summer REU/G group leader, 2015, 2016, 2017, 2018, 2023  
 Advisor of high school student research, 2015-16  
 Honors Thesis advising, 2015-16, 2018-19, 2023-24

*Outreach*

YouTube channel *Proof of Concept*, 230K+ views, 6K+ subscribers  
 Numberscope (web tool for the general public)  
 Art Exhibit: Algebraic Number Starscapes, Iceland, 2020  
 Math Club, Colorado Academy, 2018  
 CU Science Ambassador, 2016  
 Julia Robinson Math Festival, 2012  
 Workshop Leader, A Taste of Pi, 2010

*Supervision*

Postdoctoral: 2021-2024, James Rickards; 2014-2016, T. Alden Gassert  
 Ph.D.: 2022, Sarah Arpin, 2020: Daniel Martin, Hanson Smith; 2019: Robert  
 Hines; 2014: Amy Feaver  
 M.A.: 2016, Elizabeth Parsons

March 30, 2024