Modal logics, coalgebraically

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Abstract

Modal logics enrich propositional logic with further unary operators, often called box $\Box$ or diamond $\Diamond$, where usually $\Diamond\theta = \neg\Box\neg\theta$. Generalized modalities attach multiplicities, weights, probabilities, etc., to the modal operators, or even introduce binary/multi-ary modalities.

The classical $\Box/\Diamond$- semantics is given by Kripke-Structures, using unary predicates p and a non-deterministic successor relation $\longrightarrow$ with

$$a \models \Box\theta \;:\Longleftrightarrow\; \forall b. a \longrightarrow b \implies b \models \theta.$$

The famous theorem of Hennessy and Milner states that the standard $\Box/\Diamond$-logic is correct and complete for Kripke-Structures in the sense that two states are behaviourally equivalent iff they satisfy the same modal formulae. Neighbourhood systems provide a more general semantics, where monotonicity ($\frac{\theta \implies \psi}{\Box\theta \implies \Box\psi}$) need not hold.

The development of universal coalgebra, and coalgebraic modal logic in particular, has revealed that the famous Hennessy-Milner result ist just an instance of a much more general family of results. Given a modal logic, the key is to determine the appropriate type functor determining a class of coalgebras, in which the logic can be interpreted. If this type-functor satisfies some mild (natural) conditions, a corresponding Hennessy-Milner theorem can be established.