

From the pigeon hole principle to topological Ramsey spaces

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Abstract

This is a talk about the development of Ramsey theory, from combinatorial results like the classical pigeon hole principle, Ramsey's theorem [14] and Hindman's theorem [8], making emphasis in the notion of *topological Ramsey space*. In [6], Ellentuck gave a characterization of the Ramsey property which gave rise to the topological Ramsey theory. It was anticipated by the works of Nash-Williams [13], Galvin-Prikry [7] and Silver [15]. After Ellentuck's theorem, similar results were proven in different contexts (see for instance [1], [3], [9] or [16]). Each of these Ellentuck-like theorems deals with a topological Ramsey space, endowed with a convenient topology which is useful to obtain a similar characterization of the corresponding Ramsey property. The theory of topological Ramsey spaces and some of its applications have been condensed by Carlson-Simpson [2] and especially by Todorcevic [16]. Some recent developments can be seen in [4, 5, 10, 11, 12].

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