Universes.

(1) (Closure under Boolean operations) If $A, B \in \mathcal{U}$, then so are $A \cup B, A \cap B, A - B$.

(2) (Closure under products) If $A, B \in \mathcal{U}$, then so are
- $A \times B$,
- the (graphs of) the functions $\pi_1 \colon A \times B \to A$ and $\pi_2 \colon A \times B \to B$,
- $\pi_i(C)$, for any $C \in \mathcal{U}$,
- $\Delta_2(A) := \{(a, a) \mid a \in A\}$.

(3) (Closure under finite subsets) If $A \in \mathcal{U}$, then so is $\{a\}$ for any $a \in A$.

(4) (Closure under factorization) If $A, E \in \mathcal{U}$, where E is an equivalence relation on A, and $\nu \colon A \to A/E$ is the natural map, then then $A/E, \nu \in \mathcal{U}$

Properties. Assume $A, B, C, f \colon A \to B$ are in $\mathcal{U}$.

(1) If both $A_1 \subseteq A$ and $B_1 \subseteq B$ are in $\mathcal{U}$. Then $f^{-1}(B_1), f|_{A_1}, f(A_1) \in \mathcal{U}$.

(2) The set $\{(a, b, a) \mid a \in A, b \in B\} \in \mathcal{U}$.

(3) $\pi_2 \colon A \times B \times C \to B$ is in $\mathcal{U}$.

(4) $\pi_{13} \colon A \times B \times C \to A \times C$ is in $\mathcal{U}$.

(5) $s \colon A \times B \to B \times A \colon (x, y) \mapsto (y, x)$ is in $\mathcal{U}$.

(6) If f is a bijection in $\mathcal{U}$, then f^{-1} is also a bijection in $\mathcal{U}$.

(7) If $f \colon A \to B$ and $g \colon B \to C$ are in $\mathcal{U}$, then $g \circ f \colon A \to C$ is in $\mathcal{U}$.

(8) The i-th projection $\pi_i \colon A^n \to A$ is in $\mathcal{U}$.

(9) The function $s_{23} \colon A \times B \times C \to A \times C \times B \colon (a, b, c) \mapsto (a, c, b)$ is in $\mathcal{U}$.

(10) If $f \colon A \to B$ and $g \colon A \to C$ are in $\mathcal{U}$, then $f \times g \colon A \to B \times C \colon a \mapsto (f(a), g(a))$ is in $\mathcal{U}$.

(11) If $f, g \colon A \mapsto B$ are in $\mathcal{U}$, then their equalizer $\{a \in A \mid f(a) = g(a)\}$ is in $\mathcal{U}$.

(12) If $\mathbf{M}$ is an L-structure such that in $\mathcal{U}$, whose defining operations and relations are in $\mathcal{U}$, then any set first-order interpretable in $\mathbf{M}$ is in $\mathcal{U}$.