Selective and Ramsey for $\mathcal{R}_1$ ultrafilters and their Dedekind cuts

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Abstract

Motivated by a Tukey classification problem N. Dobrinen and S. Todorcevic, in [2], developed the topological Ramsey space $\mathcal{R}_1$. In [2] it is shown that the space $\mathcal{R}_1$ satisfies a canonization theorem for equivalence relations on fronts on $\mathcal{R}_1$. This extends the Pudlak-Rödl Theorem canonizing equivalence relations on the Ellentuck space.

In [3], J. Mijares associates to each topological Ramsey space $\mathcal{R}$ a notion of selective for $\mathcal{R}$ and Ramsey for $\mathcal{R}$ ultrafilter. If $\mathcal{R}$ is taken to be the Ellentuck space then the two definitions are equivalent since they reduce to the well-know equivalent notions of selective and Ramsey ultrafilter, respectively. If $\mathcal{R}$ is taken to be $\mathcal{R}_1$ then the notion of selective for $\mathcal{R}$ and Ramsey for $\mathcal{R}$ are not equivalent.

In [2], N. Dobrinen and S. Todorcevic, associate to the space $\mathcal{R}_1$ a Ramsey for $\mathcal{R}_1$ ultrafilter $\mathcal{U}_1$. They then use the canonization theorem for equivalence relations on fronts on $\mathcal{R}_1$ to completely classify all Rudin-Keisler equivalence classes of ultrafilters which are Tukey reducible to $\mathcal{U}_1$.

In the first part of this talk, we will define the space $\mathcal{R}_1$ and the notions of selective for $\mathcal{R}_1$ and Ramsey for $\mathcal{R}_1$. We will prove Proposition 5.8(2) from [2]; every Ramsey for $\mathcal{R}_1$ ultrafilter is also a weakly-Ramsey ultrafilter. We will then prove that there is a selective for $\mathcal{R}_1$ ultrafilter which is not weakly-Ramsey, a result from [4]. In particular, we will prove two theorems from [4] providing equivalent formulations of the definition of selective for $\mathcal{R}_1$ and Ramsey for $\mathcal{R}_1$. The space $\mathcal{R}_1$ comes equipped with a special function π that is finite-to-one on every element of $\mathcal{R}_1$ but never one-to-one nor constant on any element of $\mathcal{R}_1$.

In [1], Andreas Blass associates to each function $f : X \to Y$ and each ultrafilter $\mathcal{U}$ on X, a Dedekind cut in the ultrapower $\omega^Y/f(\mathcal{U})$. In [1] assuming CH, Andreas Blass has characterized, in terms of simple closure conditions, the cuts obtainable in this manner when $\mathcal{U}$ is taken to be a weakly-Ramsey ultrafilter on ω.

In the second part of this talk, assuming CH, we will characterize the cuts obtainable when $\mathcal{U}$ is taken to be a selective for $\mathcal{R}_1$ or a Ramsey for $\mathcal{R}_1$ ultrafilter and f is taken to be the map π, a result from [4]. In the remaining time, we will discuss similar results for other cuts and spaces.

References

[1] Andreas Blass. Ultrafilter mappings and their Dedekind cuts. *Transactions of the American Mathematical Society*, 188:327–340, 1974.

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