Practice with coimages, kernels, and Kernels

(1) Write down the coimage, kernel, and Kernel of the endomorphism

$$h\colon C_4 \to C_4 \colon r \mapsto r^2$$

(coimage) $\operatorname{coim}(h) = \{\{1, r^2\}, \{r, r^3\}\}$.

(kernel) $\ker(h) = \{(1,1), (1,r^2), (r^2,1), (r^2,r^2)(r,r), (r,r^3), (r^3,r), (r^3,r^3)\}$.

(Kernel) $\operatorname{Ker}(h) = \{1, r^2\}$.

(2) Write the Kernels of the following homomorphisms.

(a) $h\colon \langle \mathbb{Z}; +, -, 0\rangle \to \langle C_n; \cdot, ^{-1}, 1\rangle \colon 1 \mapsto r$

$\operatorname{Ker}(h) = n\mathbb{Z}$.

(b) $\det\colon \mathrm{GL}_n(\mathbb{R}) \to \mathbb{R}^\times \colon M \mapsto \det(M)$.

$\operatorname{Ker}(\det) = \mathrm{SL}_n(\mathbb{R})$.

(c) $A\colon \langle \mathbb{R}^\times; \cdot, ^{-1}, 1\rangle \to \langle \mathbb{R}^\times; \cdot, ^{-1}, 1\rangle \colon r \mapsto |r|$.

$\operatorname{Ker}(A) = \{\pm 1\}$.

(d) $B\colon \langle \mathbb{R}^\times; \cdot, ^{-1}, 1\rangle \to \langle \mathbb{R}^\times; \cdot, ^{-1}, 1\rangle \colon r \mapsto \operatorname{sign}(r)\ \ (\in \{\pm 1\})$.

$\operatorname{Ker}(B) = \mathbb{R}_{>0}$.

(e) $h\colon \langle \mathbb{R}; +, -, 0\rangle \to \langle \mathbb{C}^\times; \cdot, ^{-1}, 1\rangle \colon \theta \mapsto e^{i\theta}$.

$\operatorname{Ker}(B) = 2\pi\mathbb{Z}$.