HISTORY (MATH 4820): Some Practice Problems 2.

(1) State
  (a) the Chinese Remainder Theorem.
  (b) the Cardano Formula for cubics of the form $x^3 = px + q$.
  (c) Desargues' Theorem.
  (d) Bézout's Theorem.
  (e) Wallace-Bolyai-Gerwien Theorem,
  (f) Hilbert's First Problem.
  (g) Hilbert's Third Problem.
  (h) Hilbert's Seventh Problem.
  (i) Hilbert's Tenth Problem.

(2) Define the following.
  (a) congruence modulo n.
  (b) root of unity.
  (c) projective plane.
  (d) homogeneous polynomial equation.
  (e) rational number, algebraic number, and transcendental number.
  (f) ordinal number, cardinal number.

(3) Identify one mathematical reason why each of the following is remembered.
  (a) Abel.
  (b) Bolyai.
  (c) Cohen.
  (d) Dehn.
  (e) Ferrari.
  (f) Gödel.
  (g) Hilbert.
  (h) Lindemann.
  (i) Matiyasevich.
  (j) Wallace.

(4) Show how to use the Cardano formula to find the roots of $x^3 - 3x - 2 = 0$.

(5) How many points of intersection are there between the line $y = 0$ and the line at infinity in $\mathbb{RP}^2$. Illustrate the situation with a sketch.

(6) Consider the two plane curves defined by $xy = 1$ and $y = 0$.
  (a) Find all points of intersection, including points at infinity (if any).
  (b) State the intersection multiplicity at each point of intersection.

(7) Explain why $\log_3(5)$ is transcendental.