

History of Mathematical Ideas Quiz 7

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

In the 13th century CE, Cheng Dawei mentions “Han Xin’s way of counting soldiers”.¹ Cheng Dawei states that Han Xin started a battle with 1500 soldiers and lost 400-500 during the battle. To determine the exact number left, he had them stand in rows.

When the soldiers stood 3 in a row, there were 2 soldiers left over. When they lined up 5 in a row, there were 4 soldiers left over. When they lined up 7 in a row, there were 6 soldiers left over. Han Xin immediately said, “There are x soldiers”.

(I replaced the actual number Cheng Dawei attributes to Han Xin with “ x ”.)

1. Write down the congruences that should be solved to determine the number of soldiers.

$$\begin{aligned}x &\equiv -1 \pmod{3} \\x &\equiv -1 \pmod{5} \\x &\equiv -1 \pmod{7}\end{aligned}$$

2. If your congruences are each written in the form $x \equiv -1 \pmod{m_i}$, then it is possible to see how Han Xin could solve the system so quickly. What is the unique solution x to the system where x lies in the appropriate range (1000-1100 soldiers)?

The congruences of Problem 1 above can be combined into the single congruence

$$x \equiv -1 \pmod{3 \cdot 5 \cdot 7}$$

or $x \equiv -1 \pmod{105}$. The solutions are $x = -1 + 105k$ for $k \in \mathbb{Z}$. If $k = 10$, then $x = -1 + 105 \cdot \underline{10} = -1 + 1050 = 1049$ is a solution. This is the only possible solution in the interval $[1000, 1100]$ because the interval length (100) is less than the modulus (105).²

¹Han Xin was a general who lived circa 200 BCE.

²That is, the number $-1 + 105 \cdot \underline{9}$ is too small to be in $[1000, 1100]$ and the number $-1 + 105 \cdot \underline{11}$ is too large to be in $[1000, 1100]$.