History of Mathematical Ideas
Quiz 6 = Community Quiz 1!

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

The goal of this project is to use continued fractions to solve the following instance of Bézout's Identity:

$$13x + 31y = 1.$$

1. Find the continued fraction expansion of $\frac{31}{13}$.

Hint: first write $\frac{31}{13}$ as $a_0 + \frac{b_0}{c_0}$ where $a_0 \in \mathbb{Z}$ and $b_0, c_0 \in \mathbb{Z}_{>0}$ and $b_0 < c_0$. Then find $[a_0; a_1, a_2, \ldots, a_n]$ by computing:

$$a_0 + \frac{b_0}{c_0} = a_0 + \frac{1}{\frac{c_0}{b_0}} = a_0 + \frac{1}{a_1 + \frac{b_1}{c_1}} = a_0 + \frac{1}{a_1 + \frac{1}{\frac{c_1}{b_1}}} = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{b_2}{c_c}}} = \cdots$$

$\frac{31}{13} = [2; 2, 1, 1, 2]$.

2. Find the second-to-last convergent of the continued fraction from Problem 1, (i.e., evaluate $[a_0; a_1, \ldots, a_{n-1}]$), and use numerator and denominator to solve $13x + 31y = 1$.

The second to last convergent is $[2; 2, 1, 1] = \frac{12}{5}$. The relationship between the second-to-last convergent $\frac{h_{n-1}}{k_{n-1}}$ and the last convergent $\frac{h_n}{k_n}$ is

$$h_n k_{n-1} - h_{n-1} k_n = (-1)^{n-1}.$$

For us, this is

$$31 \cdot 5 - 12 \cdot 13 = (-1)^3 = -1.$$

After multiplying through by -1 this may be rewritten

$$13 \cdot 12 + 31 \cdot (-5) = 1,$$

so $(x, y) = (12, -5)$ is a solution to $13x + 31y = 1$.

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1. State Brahmagupta's Identity.

$$(x_1^2 - Ny_1^2)(x_2^2 - Ny_2^2) = (x_1x_2 + Ny_1y_2)^2 - N(x_1y_2 + x_2y_1)^2$$

2. Find the continued fraction expansion of $\dfrac{n}{n+1}$ where n is a positive integer. Write your answer in the form $[a_0; a_1, a_2, \ldots, a_k]$.

The inverse, $\left(\frac{n}{n+1}\right)^{-1} = \left(\frac{n+1}{n}\right) = 1 + \frac{1}{n}$, has continued fraction $[1; n]$, so the number itself $\left(\frac{n+1}{n}\right)^{-1} = \frac{1}{1+\frac{1}{n}}$ must have continued fraction $[0; 1, n]$.