History of Mathematical Ideas
Quiz 2

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

(1) Use the Euclidean algorithm to find $\gcd(12, 21)$.

We perform the Euclidean Algorithm:
$$\begin{aligned} \underline{21} &= 1 \cdot \underline{12} + \underline{9} \\ \underline{12} &= 1 \cdot \underline{9} + \underline{3} \\ \underline{9} &= 3 \cdot \underline{3} + \underline{0} \end{aligned}$$

and determine that the last nonzero remainder is 3 ($= \gcd(21, 12)$). (The remainder sequence is $(21, 12, 9, 3)$.)

(2) Express $\gcd(12, 21)$ in the form $12x + 21y$, where x and y are integers.

Using Back Substitution, we find that
$$\begin{aligned} \underline{3} &= \underline{12} - 1 \cdot \underline{9} \\ &= \underline{12} - 1 \cdot (\underline{21} - 1 \cdot \underline{12}) = 2 \cdot \underline{12} - 1 \cdot \underline{21}. \end{aligned}$$

This shows that for $(x, y) = (2, -1)$ we have $12x + 21y = 3 = \gcd(12, 21)$.