Practicing with the Cubic Formula and the Quartic Formula

(1) Find the depressed form of the cubic equation $x^3 + 3x^2 + 6x + 2 = 0$.

(2) Use the Cardano Formula to find a root of the depressed equation from (1), then find a root of the original equation from (1).

(3) Explain why the unique real zero of the function $f(x) = x^3 + 3x - 4$ is $x = 1$. (You might use the fact that $f'(x) > 0$ for all real x.)

Now use the Cardano Formula to argue that $\sqrt[3]{\sqrt{5}+2} - \sqrt[3]{\sqrt{5}-2}$ is a real zero of f. Conclude that $1 + \sqrt[3]{\sqrt{5}-2} = \sqrt[3]{\sqrt{5}+2}$.

(4) Find one point in $\mathbb{R}^2$ that lies on both $y = x^3 + 10$ and $xy = -3$.