Practicing with Euclidean Fields

Let $\mathbb{E}$ be any Euclidean field.

(1) Show that if $(a, b), (c, d) \in \mathbb{E}^2$ are distinct points with coordinates in $\mathbb{E}$, then there exists an equation for the line through these points whose coefficients are in $\mathbb{E}$.

(2) Show that if two non-parallel lines have equations with coefficients in $\mathbb{E}$, then the point of intersection of these lines has coordinates in $\mathbb{E}$.

(3) Show that if (a, b) has coordinates in $\mathbb{E}$ and $r \in \mathbb{E}_{>0}$, then the circle centered at (a, b) with radius r has an equation of the form $x^2 + y^2 + Ax + By + C = 0$ with coefficients in $\mathbb{E}$.

(4) Show that if circles Γ_1 and Γ_2 both have equations of the form $x^2+y^2+Ax+By+C = 0$ with coefficients in $\mathbb{E}$ and the two circles intersect, then the coordinates of the points of intersection belong to $\mathbb{E}$. (What if this problem were about the intersection points of a circle and a line instead of a circle and a circle?)