Ordered Fields.

Definition 1. A **field** is an algebraic structure $\mathbb{F} = \langle F; +, -, 0, \cdot, 1\rangle$ which satisfies the following

(1) Additive laws[1]:
   (a) (Associative law) $\forall x \forall y \forall z((x + (y + z)) = ((x + y) + z))$.
   (b) (Commutative law) $\forall x \forall y(x + y = y + x)$.
   (c) (Unit law) $\forall x(x + 0 = x)$
   (d) (Inverse law) $\forall x(x + (-x) = 0)$
(2) Multiplicative laws:
   (a) (Associative law) $\forall x \forall y \forall z((x(yz)) = ((xy)z))$.
   (b) (Commutative law) $\forall x \forall y(xy = yx)$.
   (c) (Unit law) $\forall x(x1 = x)$
(3) Law linking addition to multiplication:
   (a) (Distributive law): $\forall x \forall y \forall z(x(y + z) = xy + xz)$.
(4) Other defining properties that are not laws:
   (a) $0 \neq 1$.
   (b) $\forall x((x \neq 0) \to \exists y(xy = 1))$.

If you know the definition of "abelian group", the axioms for fields say that $\mathbb{F}$ is additively an abelian group, $\mathbb{F} - \{0\}$ is multiplicatively an abelian group, and the additive and multiplicative structures are linked by the distributive law.

Examples. $\mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{Z}/p\mathbb{Z}$ (p prime).
Nonexamples. $\mathbb{N}, \mathbb{Z}, \mathbb{Z}/n\mathbb{Z}$ (n not a prime).

Definition 2. An **ordered field** is a structure $\mathbb{F} = \langle F; +, -, 0, \cdot, 1, \leq\rangle$ where

(1) $\langle F; +, -, 0, \cdot, 1\rangle$ is a field,
(2) $\langle F; \leq\rangle$ is a totally ordered set (which means that $\leq$ is a reflexive, antisymmetric, transitive relation satisfying $a \leq b, a = b$, or $b \leq a$ for all a, b),
(3) (Order structure is linked to field structure):
   (a) (Additive compatibility) $\forall x \forall y \forall z((y \leq z) \to (x + y \leq x + z))$
   (b) (Multiplicative compatibility) $\forall x \forall y \forall z(((y \leq z) \wedge (0 \leq x)) \to (xy \leq xz))$

Examples. $\mathbb{Q}, \mathbb{R}$.
Nonexamples. $\mathbb{C}, \mathbb{Z}/n\mathbb{Z}$ for any n.

[1]A **law** or **identity** is a universally quantified equation.