

The Cardano Formula

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Thus, if (given specific values for p and q) we can solve for u and v in the system

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then we can set $y = u + v$ to solve $y^3 = py + q$.

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- ② Find two numbers in double proportion $[x, 2x]$ such that if the square of the larger is multiplied by the lesser and to the product is added the sum of the numbers, the result is 40.

$$(2x)^2x + x + 2x = 40, \quad \text{or} \quad 4x^3 + 3x - 40 = 0. \quad \text{Find } x \text{ and } 2x.$$

- ③ Find the number which added to its cube gives 5.

$$x + x^3 = 5. \quad \text{or} \quad x^3 + x - 5 = 0.$$