ABSTRACT ALGEBRA 1 (MATH 3140): Practice Problems 2

Sample Problems.

(1) Define the following:
  (a) cycle type of a permutation.
  (b) subgroup lattice.
  (c) simple group.
  (d) a complement of a subgroup.
  (e) the normalizer of a subgroup.
  (f) the direct product of two groups.
  (g) the semidirect product of two groups.
  (h) elementary divisor.
  (i) invariant factor.

(2) Give examples of the following:
  (a) a group that has a proper direct product decomposition.
  (b) a group that has a proper semidirect product decomposition.
  (c) a nonabelian simple group.
  (d) a composition series for S_3.
  (e) a finite group whose subgroup lattice is a chain of three elements.
  (f) a number n such that there exactly three isomorphism types of abelian groups of size n.
  (g) a finite abelian group that is not simple which has only one elementary divisor.

(3) State
  (a) the characterization of products.
  (b) the characterization of semidirect products.
  (c) the Jordan-Hölder Theorem.
  (d) the Second Isomorphism Theorem.
  (e) the Jordan-Hölder Theorem.
  (f) the Schur-Zassenhaus Theorem.
  (g) the Third Isomorphism Theorem.
  (h) the Correspondence Theorem.

(4) Which elements of S_n have the largest Cauchy number?

(5) Give one representative of each conjugacy class of S_5.

(6) How many isomorphism types of direct products are there of the form $\mathbb{Z}_3 \times \mathbb{Z}_4$? How many isomorphism types of semidirect products are there of the form $\mathbb{Z}_3 \rtimes \mathbb{Z}_4$?

(7) How many isomorphism types are there of abelian groups of size 200?

(8) Assume that we are dealing with groups. Show that if $A \cong C$ and $B \cong D$, then $A \times B \cong C \times D$.

(9) What are the elementary divisors of C_n if $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ and $p_1 < p_2 < \cdots < p_k$ are prime?