

ABSTRACT ALGEBRA 1 (MATH 3140): Practice Problems

Sample Problems.

- (1) Define the following:
 - (a) arity of a function.
 - (b) coimage of a function.
 - (c) permutation.
 - (d) abelian group.
 - (e) cyclic group.
 - (f) dihedral group.
 - (g) homomorphism.
 - (h) subgroup.
 - (i) index of a subgroup.
- (2) Give examples of the following:
 - (a) a semigroup that is not a group.
 - (b) a group that is not a semigroup.
 - (c) a group of order 97.
 - (d) an abelian group that is not cyclic.
 - (e) a homomorphism that is not an isomorphism.
 - (f) an isomorphism that is not an automorphism.
- (3) State
 - (a) the Cayley Representation Theorem.
 - (b) the First Isomorphism Theorem.
 - (c) Lagrange's Theorem.
- (4) Describe all homomorphisms $h: \mathbb{Z} \rightarrow \mathbb{Z}$. What are the kernels and images?
- (5) Explain why the multiplication table of a group is a Latin square.
- (6) Suppose that h be a homomorphism between finite groups and that
 - (a) $a = |\text{dom}(h)|$.
 - (b) $b = |\text{cod}(h)|$.
 - (c) $c = |\text{im}(h)|$.
 - (d) $d = |\text{coim}(h)|$.
 - (e) $e = |\text{Ker}(h)|$.

Write down all divisibility relationships that are guaranteed to hold among a, b, c, d, e for any choice of h .

- (7) Explain why S_n is isomorphic to a subgroup of S_{n+1} .
- (8) What is the kernel of determinant function $\text{GL}_n(\mathbb{R}) \rightarrow \mathbb{R}^\times$?
- (9) Let G be a group with subgroups H and K and assume that K is normal in G . Show that the set $HK = \{hk \mid h \in H, k \in K\}$ is the smallest subgroup of G containing both H and K .