Abstract Algebra 1
Quiz 9

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. It is a fact that if $\mathbf{1}$ is the one element group and G is any group, then

   - $\mathbf{1} \times G \cong G$, and
   - $G \times \mathbf{1} \cong G$.

   Write down the maps that you would use to establish these isomorphisms.

   For the isomorphism in the first bullet point, use $\pi_2 \colon \mathbf{1} \times G \to G \colon (1, g) \mapsto g$. (This is the second coordinate projection map.)
   For the isomorphism in the first bullet point, use $\pi_2 \colon G \times \mathbf{1} \to G \colon (g, 1) \mapsto g$. (This is the first coordinate projection map.)

2. It is a fact that if G, H, K are groups, then

   - $G \times (H \times K) \cong (G \times H) \times K$.

   Write down the map that you would use to establish this isomorphism.

   For this isomorphism, use $h \colon G \times (H \times K) \to (G \times H) \times K \colon (g, (h, k)) \mapsto ((g, h), k)$.

The isomorphisms indicated above show that the direct product operation satisfies (up to isomorphism) those identities that define monoids. That is, the direct product operation is associative (up to isomorphism) and has an identity element (up to isomorphism).