Abstract Algebra 1
Quiz 6

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

Let G be a group, let S be a subgroup of G, and let $a, b \in G$.

1. Explain why $a \in aS$.

Since $1 \in S$ we have $a = a \cdot 1 \in aS$.

2. Explain why the cosets aS and bS are either disjoint or equal.

Assume that aS and bS are not disjoint. For example, assume that $c \in aS \cap bS$. There must exist $u, v \in S$ such that $au = c = bv$. We now argue that $aS \subseteq bS$. Choose any $d \in aS$, say $d = aw$ for some $w \in S$. Then $d = aw = (bvu^{-1})w = b(vu^{-1})w) \in bS$ since $vu^{-1}w \in S$. This shows that $aS \subseteq bS$. A symmetric argument shows that $bS \subseteq aS$, so $aS = bS$. Altogether we have shown that if aS and bS are not disjoint, then they are equal.