Abstract Algebra 1
Quiz 2

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Let $S = \{a, b, c, \dots\}$ be a set of more than one element and let $x \circ y$ be *left projection* on S. (This means that $(\forall x)(\forall y)(x \circ y = x)$.) Show that $\langle S; \circ\rangle$ is a right cancellative semigroup that is not left cancellative.

There are three things to show:

(1) $\circ$ is associative.
(2) $\langle S; \circ\rangle$ is right cancellative.
(3) $\langle S; \circ\rangle$ is not left cancellative.

For the first item, for any $x, y, z \in S$ we have

$$x \circ (y \circ z) = x \circ y = x = (x \circ y) = (x \circ y) \circ z,$$

so the multiplication is associative.

For the second item, assume that for some $p, q, r \in S$ we have $p \circ r = q \circ r$. Then $p = p \circ r = q \circ r = q$, so $p = q$. Thus, we can always cancel r from the right in any equality ending in r.

For the third item, choose $p, q, r \in S$ with $p \neq q$. Then $r \circ p = r = r \circ q$, but $p \neq q$, so we cannot cancel the r from the left in this equality starting with r.