

# Union and Intersection



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Worth remembering: The Axiom of Separation only allows us to take the intersection of a nonempty collection.