The First Axioms of Set Theory

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For all x and forall y, x is equal to y if and only if, for all z, $\underbrace{z \text{ belongs to } x \text{ iff } z \text{ belongs to } y}_{x \text{ and } y \text{ have the same elements } z}$.

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The y in this axiom is called the union of x. We write $y = \bigcup x$.

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$$(\forall x)(\exists y)(\forall z)((z \in y) \leftrightarrow (\exists w)(z \in w) \land (w \in x)). \qquad (\land = \text{and})$$

Example.
If $x = \{\{A, B, C\}, \{C, D\}, \{D, E\}\}$, then $\bigcup x = \{A, B, C, D, E\}$.

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If x is a set, then there is a set $\mathcal{P}(x)$ whose elements are exactly the subsets of x.

$$(\forall x)(\exists P)(\forall y)((y \in P) \leftrightarrow (y \subseteq x)).$$

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