Propositional logic

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- W = "The ground is wet".

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	R	W	$R \wedge W$		R	W	$R \lor$	W			
	0	0	0		0	0	0		_	$R \parallel \neg R$	
	0	1	0		0	1	1			0 1	
	1	0	0		1	0	1			1 0	
	1	1	1		1	1	1				
W	R -	$\rightarrow W$	R	W	R	$\leftrightarrow W$		R	W	$R \oplus W$ =	$= R \vee$
0		1	0	0		1		0	0	()

 $1 \quad 0$

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This proposition P is a **tautology**, because it assumes the value "true" under any truth assignment to the propositional variables. This means that P is true because of its logical structure alone, and not because of the truth values of its variables.

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Two propositions are (**logically**) equivalent if they assume the same truth value under any truth assignment to the propositional variables. (Write $P \equiv Q$.)

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- $\neg(\neg P) \equiv P$,
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- $H \to C$ is logically independent of $C \to H$.
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H	C	$\neg H$	$\neg C$	$H \to C$	$(\neg C) \rightarrow (\neg H)$	$C \to H$	$(\neg H) \to (\neg C)$
0	0	1	1	1	1	1	1
0	1	1	0	1	1	0	0
1	0	0	1	0	0	1	1
1	1	0	0	1	1	1	1
				direct	contrapositive	converse	inverse

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Note that the inverse is the contrapositive of the converse,

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Note that the inverse is the contrapositive of the converse, hence it is logically independent of the original implication.

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0	1	1	0	1	1	0	0
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1	1	0	0	1	1	1	1
				direct	contrapositive	converse	inverse

Note that the inverse is the contrapositive of the converse, hence it is logically independent of the original implication.

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H	C	$\neg H$	$\neg C$	$H \to C$	$(\neg C) \rightarrow (\neg H)$	$C \to H$	$(\neg H) \rightarrow (\neg C)$
0	0	1	1	1	1	1	1
0	1	1	0	1	1	0	0
1	0	0	1	0	0	1	1
1	1	0	0	1	1	1	1
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0	1	1	0	1	1	0	0
1	0	0	1	0	0	1	1
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"If we adopt the new policy, then things will get better."

$$H$$
 C

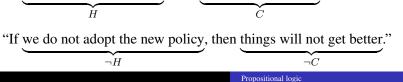
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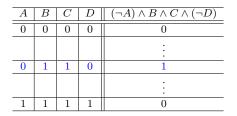
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The truth table of a monomial has exactly one row whose value is T = 1:



The monomial $(\neg A) \land B \land C \land (\neg D)$ assumes value 1 iff A = 0, B = 1, C = 1, D = 0.

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Small example. Create a proposition with truth table

A	B	C	?
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
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Theorem. Every propositional formula is logically equivalent to a formula in $DNF = \bigvee (\land \pm \text{ variables}).$

Corollary. The symbols \land, \lor, \neg are a "complete" set of logical connectives, in the sense that any proposition is logically equivalent to one expressed with $\{\land,\lor,\neg\}$ + propositional variables.

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 $(A \land (\neg B) \land (\neg C)) \lor (A \land (\neg B) \land C) \lor (A \land B \land (\neg C)) \lor (A \land B \land C)$

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0	1	1	1
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1	0	1	1
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 $\begin{array}{ccc} ((\neg p) \land (\neg q) \land (\neg r)) \lor ((\neg p) \land (\neg q) \land r) \lor ((\neg p) \land q \land (\neg r)) \\ \lor & ((\neg p) \land q \land r) \lor (p \land (\neg q) \land r) \lor (p \land q \land r) \end{array}$