Practice with quantifiers!

(1) In this problem you will put "$((\neg((\forall x)(x = 0))) \wedge (\neg((\forall x)(x \neq 0))))$" in prenex form.

(a) Draw a formula tree for the statement.

(b) Standardize the variables apart.

(c) Put in prenex form.

(2) Determine the truth of the statement in the previous problem in $\mathbb{R}$ by giving a winning strategy for the appropriate quantifier.

(3) Let $f : A \to B$ be a function, and consider the structure $\langle A, B; f \rangle$. Write down a formal sentence, that is meaningful for this structure, and which expresses "f is a surjective function".

(4) For each of the following instances of the previous problem, give a winning strategy for the appropriate quantifier:

(a) $A = B = \mathbb{R}$, $f(x) = x^3$.

(b) $A = B = \mathbb{R}$, $f(x) = e^x$.

(5) Write "$(\forall x)(\forall y)((x < y) \to (\exists z)(x < z < y))$" in prenex form. Is the resulting sentence true in $\mathbb{R}$? in $\mathbb{N}$?

(6) Write the following in prenex form.

(a) The Axiom of Extensionality.

(b) The Axiom of Pairing.

(c) The Axiom of Union.

(7) Is $(\forall a)(\exists b)(\forall c)(\exists d)(a^2 + b^2 = c^2 + d^2)$ true in $\mathbb{R}$? in $\mathbb{C}$? For each structure, give a winning strategy for the appropriate quantifier.