Meaning

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$+$	0	1	2
0	0	1	2
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2	2	0	1

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2	0	2	1

$<$	0	1	2
0	F	T	T
1	F	F	T
2	F	F	F

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$<$	0	1	2
0	F	T	T
1	F	F	T
2	F	F	F

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α	0	1	2
0	F	F	F
1	F	T	T
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$\Longrightarrow$

$(\exists y)\alpha$	0	1	2
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0	F	F	F
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α	0	1	2
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$\Longrightarrow$

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
2	T	T	T

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
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$\Longrightarrow$

$(\forall x)(\exists y)\alpha$	0	1	2
0	F	F	F
1	F	F	F
2	F	F	F

Conclusion:

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α	0	1	2
0	F	F	F
1	F	T	T
2	F	T	T

$\Longrightarrow$

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
2	T	T	T

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
2	T	T	T

$\Longrightarrow$

$(\forall x)(\exists y)\alpha$	0	1	2
0	F	F	F
1	F	F	F
2	F	F	F

Conclusion: The sentence is false in $\mathbf{A}$.