Induction proofs: examples and nonexamples

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$$\begin{aligned} 1 + 2 + \cdots + n + (n+1) &= n(n+1)/2 + (n+1) \\ &= n(n+1)/2 + 2(n+1)/2 \\ &= (n+1)[n/2 + 2/2] \\ &= (n+1)(n+2)/2, \end{aligned}$$

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Exercise.

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Exercise. Prove $1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$ in a similar way.

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instead of to the sequence S_n. (I.e., Prove T_0, then prove $T_n \rightarrow T_{n+1}$.)

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Inductive Step.

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Case 1.

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In this case, $n + 1$ is a product of one prime.

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$$N_k = \binom{k}{4} + \binom{k-1}{2} + \binom{k}{1} = \frac{k^4 - 6k^3 + 23k^2 - 18k + 24}{24}.$$

Mistaken induction proofs, 2

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What is the mistake?

We did not prove the Base Case. (In fact, the statement is false.)

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Inductive Step. Assume S_n is true

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Base Case. $(n = 0)$ If the list is (h_0), with one horse only, then all horses in the list have the same color. ✓

Inductive Step. Assume S_n is true and prove S_{n+1}.

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Corollary.

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Corollary. All horses are white.

Mistaken induction proofs, 4

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Theorem.

Theorem. π is rational.

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Mistaken induction proofs, 4

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We only proved that all finite decimal approximations π are rational, but not that the limit π is rational.