Homomorphisms

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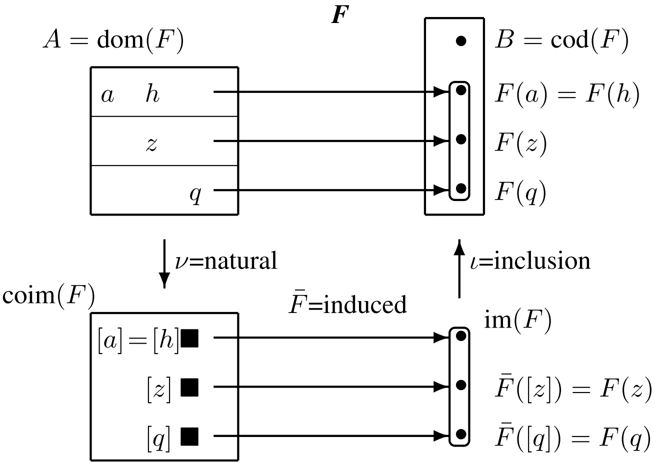

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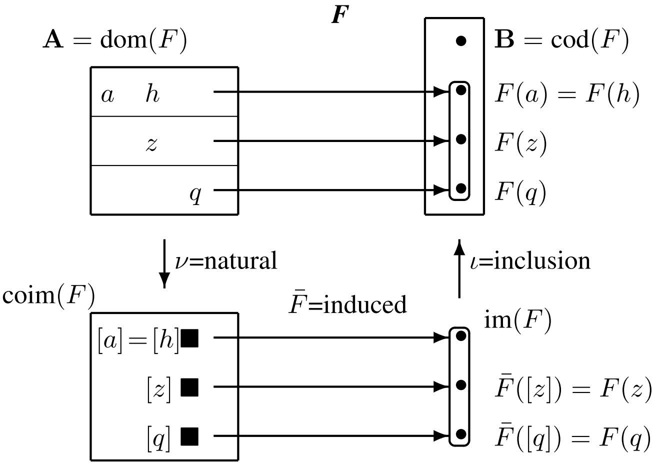

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By this, I mean that if $\mathbf{A}$ is a structure of some sort and $\mathrm{Aut}(\mathbf{A})$ is the collection of automorphisms of $\mathbf{A}$, then $\mathrm{Aut}(\mathbf{A})$ usually has a group structure, since

1. the composition of two automorphisms is usually an automorphism,
2. every automorphism has an inverse automorphism,
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