Discrete Math
Quiz 9

Name:____________________________

You have 10 minutes to complete this quiz. You may not use any unauthorized sources and you may not communicate with others about the exam. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Prove Theorem A, a statement about the natural numbers, using one of the following strategies. Circle the strategy that best describes the strategy you are using. There are many possible answers to this question!

Direct Proof   Proof of the Contrapositive   Proof by Contradiction

Theorem A. If n is even and $n = k^2$, then k is even.

Proof.

Assume that (i) n is even, (ii) $n = k^2$, and (iii) k is not even.

From Item (iii) and the definition at the foot of the page we know that k is odd. By the theorem at the foot of the page we know that $k = (2m + 1)$ for some natural number m. From Item (ii), we get the first equality in:

$$n = k^2 = (2m + 1)^2 = 2(2m^2 + 2m) + 1.$$

Let $M = 2m^2+2m$. By the theorem at the foot of the page, the fact that $n = 2M+1$ implies that n is odd. By the definition at the foot of the page, the fact that n is odd implies that n not even. This contradicts Item (i).

□

You may use the following definitions and theorem in your proof.

Definition. A natural number n is **even** if there exists some natural number m such that $n = 2m$. A natural number that is not even is called **odd**.

Theorem. A natural number n is odd if and only if there exists a natural number m such that $n = 2m + 1$.