Practice Problems.

1. Give an indirect proof of "There is a unique set with no elements." Then explain why statements of the form "There is a unique X with property Y" should be easier to prove indirectly than directly.

2.

(a) Prove or disprove: "If f is increasing and g is increasing, then $f \circ g$ is increasing."

(b) Prove or disprove: "If f is decreasing and g is decreasing, then $f \circ g$ is decreasing."

3. Prove or disprove: "If f is bounded above and g is bounded above, then $f + g$ is bounded above."

4. Give examples of statements of the form $H \to C$ such that the following properties hold. If no example exists, explain why.

(a) The statement and its converse are both true.

(b) The statement is true, but the converse is false.

(c) The statement is false, but the converse is true.

(d) The statement and its converse are both false.

(e) The statement is true and its contrapositive is false.

(f) The contrapositive and the converse are both false.

(g) The contrapositive is true and the converse is false.