The Axiom of Separation

The Axiom of Separation

The Axiom of Separation

Given a set** A **and a property given by a formula** φ**, there is a set whose elements are the elements of** A **that satisfy** φ**.

The Axiom of Separation

Given a set A and a property given by a formula φ, there is a set whose elements are the elements of A that satisfy φ.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

The Axiom of Separation

Given a set A and a property given by a formula φ, there is a set whose elements are the elements of A that satisfy φ.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set.

The Axiom of Separation

Given a set A and a property given by a formula φ, there is a set whose elements are the elements of A that satisfy φ.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers.

The Axiom of Separation

Given a set** A **and a property given by a formula** φ**, there is a set whose elements are the elements of** A **that satisfy** φ**.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers. We let $\pi(x)$ be a formula that expresses "x is prime".

The Axiom of Separation

Given a set A ***and a property given by a formula*** φ***, there is a set whose elements are the elements of*** A ***that satisfy*** φ***.***

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers. We let $\pi(x)$ be a formula that expresses "x is prime". Then the set of primes is $\{x \in \mathbb{N} \mid \pi(x)\}$.

The Axiom of Separation

Given a set A and a property given by a formula φ, there is a set whose elements are the elements of A that satisfy φ.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers. We let $\pi(x)$ be a formula that expresses "x is prime". Then the set of primes is $\{x \in \mathbb{N} \mid \pi(x)\}$.

The Axiom of Separation allows us to "cut out a subset" of a known set.

The Axiom of Separation

Given a set A and a property given by a formula φ, there is a set whose elements are the elements of A that satisfy φ.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers. We let $\pi(x)$ be a formula that expresses "x is prime". Then the set of primes is $\{x \in \mathbb{N} \mid \pi(x)\}$.

The Axiom of Separation allows us to "cut out a subset" of a known set. If we can describe the subset (by φ), then it is a set.

The Axiom of Separation

Given a set A and a property given by a formula φ, there is a set whose elements are the elements of A that satisfy φ.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers. We let $\pi(x)$ be a formula that expresses "x is prime". Then the set of primes is $\{x \in \mathbb{N} \mid \pi(x)\}$.

The Axiom of Separation allows us to "cut out a subset" of a known set. If we can describe the subset (by φ), then it is a set.

Mini-Theorem. If $S = \{x \in A \mid \varphi(x)\}$, then $S \subseteq A$.

The Axiom of Separation

Given a set A and a property given by a formula φ, there is a set whose elements are the elements of A that satisfy φ.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers. We let $\pi(x)$ be a formula that expresses "x is prime". Then the set of primes is $\{x \in \mathbb{N} \mid \pi(x)\}$.

The Axiom of Separation allows us to "cut out a subset" of a known set. If we can describe the subset (by φ), then it is a set.

Mini-Theorem. If $S = \{x \in A \mid \varphi(x)\}$, then $S \subseteq A$.

Mini-Proof. We must show that $z \in S$ implies $z \in A$.

The Axiom of Separation

Given a set A ***and a property given by a formula*** φ***, there is a set whose elements are the elements of*** A ***that satisfy*** φ***.***

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers. We let $\pi(x)$ be a formula that expresses "x is prime". Then the set of primes is $\{x \in \mathbb{N} \mid \pi(x)\}$.

The Axiom of Separation allows us to "cut out a subset" of a known set. If we can describe the subset (by φ), then it is a set.

Mini-Theorem. If $S = \{x \in A \mid \varphi(x)\}$, then $S \subseteq A$.

Mini-Proof. We must show that $z \in S$ implies $z \in A$. Assume that $z \in S$.

The Axiom of Separation

Given a set A and a property given by a formula φ, there is a set whose elements are the elements of A that satisfy φ.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers. We let $\pi(x)$ be a formula that expresses "x is prime". Then the set of primes is $\{x \in \mathbb{N} \mid \pi(x)\}$.

The Axiom of Separation allows us to "cut out a subset" of a known set. If we can describe the subset (by φ), then it is a set.

Mini-Theorem. If $S = \{x \in A \mid \varphi(x)\}$, then $S \subseteq A$.

Mini-Proof. We must show that $z \in S$ implies $z \in A$. Assume that $z \in S$. By the definition of S, $z \in A$ and $\varphi(z)$ holds.

The Axiom of Separation

Given a set A and a property given by a formula φ, there is a set whose elements are the elements of A that satisfy φ.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers. We let $\pi(x)$ be a formula that expresses "x is prime". Then the set of primes is $\{x \in \mathbb{N} \mid \pi(x)\}$.

The Axiom of Separation allows us to "cut out a subset" of a known set. If we can describe the subset (by φ), then it is a set.

Mini-Theorem. If $S = \{x \in A \mid \varphi(x)\}$, then $S \subseteq A$.

Mini-Proof. We must show that $z \in S$ implies $z \in A$. Assume that $z \in S$. By the definition of S, $z \in A$ and $\varphi(z)$ holds. Already this shows that $z \in A$.

The Axiom of Separation

Given a set** A **and a property given by a formula** φ**, there is a set whose elements are the elements of** A **that satisfy** φ**.

That is, if A is a set, then $\{x \in A \mid \varphi(x)\}$ is a set.

Example. We will see later that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set. The Axiom of Separation allows us to create from $\mathbb{N}$ the set of prime numbers. We let $\pi(x)$ be a formula that expresses "x is prime". Then the set of primes is $\{x \in \mathbb{N} \mid \pi(x)\}$.

The Axiom of Separation allows us to "cut out a subset" of a known set. If we can describe the subset (by φ), then it is a set.

Mini-Theorem. If $S = \{x \in A \mid \varphi(x)\}$, then $S \subseteq A$.

Mini-Proof. We must show that $z \in S$ implies $z \in A$. Assume that $z \in S$. By the definition of S, $z \in A$ and $\varphi(z)$ holds. Already this shows that $z \in A$. $\square$

Axiom of Separation = “Restricted Comprehension”

Axiom of Separation = “Restricted Comprehension”

Let’s compare two “slightly” different versions of the Axiom of Separation.

Axiom of Separation = “Restricted Comprehension”

Let’s compare two “slightly” different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.

Axiom of Separation = “Restricted Comprehension”

Let’s compare two “slightly” different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.

Axiom of Separation = "Restricted Comprehension"

Let's compare two "slightly" different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

Axiom of Separation = "Restricted Comprehension"

Let's compare two "slightly" different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

Axiom of Separation = "Restricted Comprehension"

Let's compare two "slightly" different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

(Comprehend = "include, comprise, encompass".)

Axiom of Separation = “Restricted Comprehension”

Let’s compare two “slightly” different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

(Comprehend = “include, comprise, encompass”.)

In the second version, we “restrict” the choice of x to belong to a known set.

Axiom of Separation = “Restricted Comprehension”

Let’s compare two “slightly” different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

(Comprehend = “include, comprise, encompass”.)

In the second version, we “restrict” the choice of x to belong to a known set. In the first version, there is no restriction.

Axiom of Separation = “Restricted Comprehension”

Let’s compare two “slightly” different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

(Comprehend = “include, comprise, encompass”.)

In the second version, we “restrict” the choice of x to belong to a known set. In the first version, there is no restriction. In the first version, we can create any set as long as we can describe its members.

Axiom of Separation = “Restricted Comprehension”

Let’s compare two “slightly” different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

(Comprehend = “include, comprise, encompass”.)

In the second version, we “restrict” the choice of x to belong to a known set. In the first version, there is no restriction. In the first version, we can create any set as long as we can describe its members. We will later see why Unrestricted Comprehension is inconsistent, so we avoid its use except in the statement of the axioms.

Axiom of Separation = “Restricted Comprehension”

Let’s compare two “slightly” different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

(Comprehend = “include, comprise, encompass”.)

In the second version, we “restrict” the choice of x to belong to a known set. In the first version, there is no restriction. In the first version, we can create any set as long as we can describe its members. We will later see why Unrestricted Comprehension is inconsistent, so we avoid its use except in the statement of the axioms. That is, we allow unrestricted comprehension in the statement of the axioms, but to further develop mathematics we confine ourselves to restricted comprehension.

Axiom of Separation = "Restricted Comprehension"

Let's compare two "slightly" different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

(Comprehend = "include, comprise, encompass".)

In the second version, we "restrict" the choice of x to belong to a known set. In the first version, there is no restriction. In the first version, we can create any set as long as we can describe its members. We will later see why Unrestricted Comprehension is inconsistent, so we avoid its use except in the statement of the axioms. That is, we allow unrestricted comprehension in the statement of the axioms, but to further develop mathematics we confine ourselves to restricted comprehension.

Example.

Axiom of Separation = "Restricted Comprehension"

Let's compare two "slightly" different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

(Comprehend = "include, comprise, encompass".)

In the second version, we "restrict" the choice of x to belong to a known set. In the first version, there is no restriction. In the first version, we can create any set as long as we can describe its members. We will later see why Unrestricted Comprehension is inconsistent, so we avoid its use except in the statement of the axioms. That is, we allow unrestricted comprehension in the statement of the axioms, but to further develop mathematics we confine ourselves to restricted comprehension.

Example. The Axiom of Pairing may be thought of as an example of Unrestricted Comprehension in the axioms:

Axiom of Separation = "Restricted Comprehension"

Let's compare two "slightly" different versions of the Axiom of Separation.

- (Unrestricted Comprehension) $\{x \mid \varphi(x)\}$ is a set.
- (Restricted Comprehension) $\{x \in A \mid \varphi(x)\}$ is a set.

(Comprehend = "include, comprise, encompass".)

In the second version, we "restrict" the choice of x to belong to a known set. In the first version, there is no restriction. In the first version, we can create any set as long as we can describe its members. We will later see why Unrestricted Comprehension is inconsistent, so we avoid its use except in the statement of the axioms. That is, we allow unrestricted comprehension in the statement of the axioms, but to further develop mathematics we confine ourselves to restricted comprehension.

Example. The Axiom of Pairing may be thought of as an example of Unrestricted Comprehension in the axioms: If U and V are sets, then so is

$$\{x \mid (x = U) \vee (x = V)\}.$$

More on Unrestricted Comprehension

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension.

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.)

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.) On this page, we indicate why, intuitively, Unrestricted Comprehension might be a source of problems.

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.) On this page, we indicate why, intuitively, Unrestricted Comprehension might be a source of problems.

Example.

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.) On this page, we indicate why, intuitively, Unrestricted Comprehension might be a source of problems.

Example. Suppose I want to create the set of all mathematical concepts.

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.) On this page, we indicate why, intuitively, Unrestricted Comprehension might be a source of problems.

Example. Suppose I want to create the set of all mathematical concepts. I write down a formula $\varphi(x)$ expressing that "x is a mathematical concept",

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.) On this page, we indicate why, intuitively, Unrestricted Comprehension might be a source of problems.

Example. Suppose I want to create the set of all mathematical concepts. I write down a formula $\varphi(x)$ expressing that "x is a mathematical concept", then I define $C = \{x \mid \varphi(x)\}$.

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.) On this page, we indicate why, intuitively, Unrestricted Comprehension might be a source of problems.

Example. Suppose I want to create the set of all mathematical concepts. I write down a formula $\varphi(x)$ expressing that "x is a mathematical concept", then I define $C = \{x \mid \varphi(x)\}$. Note that the meaning of C changes over time as new mathematical concepts are created.

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.) On this page, we indicate why, intuitively, Unrestricted Comprehension might be a source of problems.

Example. Suppose I want to create the set of all mathematical concepts. I write down a formula $\varphi(x)$ expressing that "x is a mathematical concept", then I define $C = \{x \mid \varphi(x)\}$. Note that the meaning of C changes over time as new mathematical concepts are created. If C is used in the creation of new mathematical concepts, then the meaning of the new concepts also changes over time.

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.) On this page, we indicate why, intuitively, Unrestricted Comprehension might be a source of problems.

Example. Suppose I want to create the set of all mathematical concepts. I write down a formula $\varphi(x)$ expressing that "x is a mathematical concept", then I define $C = \{x \mid \varphi(x)\}$. Note that the meaning of C changes over time as new mathematical concepts are created. If C is used in the creation of new mathematical concepts, then the meaning of the new concepts also changes over time. Nothing is definite anymore, and the changing meanings might lead to contradiction.

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.) On this page, we indicate why, intuitively, Unrestricted Comprehension might be a source of problems.

Example. Suppose I want to create the set of all mathematical concepts. I write down a formula $\varphi(x)$ expressing that "x is a mathematical concept", then I define $C = \{x \mid \varphi(x)\}$. Note that the meaning of C changes over time as new mathematical concepts are created. If C is used in the creation of new mathematical concepts, then the meaning of the new concepts also changes over time. Nothing is definite anymore, and the changing meanings might lead to contradiction.

Point to Ponder.

More on Unrestricted Comprehension

Naive Set Theory allows Unrestricted Comprehension. (NST specifies that any "unordered collection of distinct objects" is a set, so anything one can write down, like $\{x \mid \varphi(x)\}$, is a set.) On this page, we indicate why, intuitively, Unrestricted Comprehension might be a source of problems.

Example. Suppose I want to create the set of all mathematical concepts. I write down a formula $\varphi(x)$ expressing that "x is a mathematical concept", then I define $C = \{x \mid \varphi(x)\}$. Note that the meaning of C changes over time as new mathematical concepts are created. If C is used in the creation of new mathematical concepts, then the meaning of the new concepts also changes over time. Nothing is definite anymore, and the changing meanings might lead to contradiction.

Point to Ponder. Why is this problem unlikely to arise if we confine ourselves to Restricted Comprehension/Separation?