Practice with propositions!

Remember the tables for $\wedge, \vee, \neg, \rightarrow, \leftrightarrow, \veebar$

p	q	$p \wedge q$
0	0	
0	1	
1	0	
1	1	

p	q	$p \vee q$
0	0	
0	1	
1	0	
1	1	

p	$\neg p$
0	
1	

p	q	$p \rightarrow q$
0	0	
0	1	
1	0	
1	1	

p	q	$p \leftrightarrow q$
0	0	
0	1	
1	0	
1	1	

p	q	$p \veebar q$
0	0	
0	1	
1	0	
1	1	

Establish the following equivalences! (You may use the back of this page.)

The goal is to show that $\{\wedge, \vee, \neg\}$ are sufficient to express all the connectives from above. Then, you are asked to show that $\{\rightarrow, \neg\}$ are also sufficient to express all the connectives from above.

(1) ($\veebar$ is redundant) $p \veebar q \equiv \neg(p \leftrightarrow q)$
(2) ($\leftrightarrow$ is redundant) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
(3) ($\rightarrow$ is redundant) $p \rightarrow q \equiv (\neg p) \vee q$

This is enough to show that any proposition expressible using $\wedge, \vee, \neg, \rightarrow, \leftrightarrow, \veebar$ can be re-expressed using only $\wedge, \vee, \neg$. Now, we claim that any proposition expressible using $\wedge, \vee, \neg$ can be re-expressed using only $\rightarrow, \neg$. Verify the following:

(4) $p \vee q \equiv (\neg p) \rightarrow q$
(5) $p \wedge q \equiv \neg(p \rightarrow (\neg q))$

Example. Let's check Item 5.

p	q	$\neg q$	$p \rightarrow (\neg q)$	$\neg(p \rightarrow (\neg q))$	$p \wedge q$
0	0	1	1	0	0
0	1	0	1	0	0
1	0	1	1	0	0
1	1	0	0	1	1