Logic

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$$(\forall x)(((x > 2) \wedge \varphi_{\text{even}}(x)) \rightarrow (\exists y)(\exists z)(\varphi_{\text{prime}}(y) \wedge \varphi_{\text{prime}}(z) \wedge (x = y + z))).$$

I would read this aloud by saying "For all x, if x is greater than 2 and satisfies a formula expressing that x is even, then there exists y and z, each satisfying a formula expressing that they are prime, such that x equals y plus z".

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$$(x > 1) \wedge (\forall u)((\exists v)(x = uv) \rightarrow ((u = 1) \vee (u = x))),$$

Goldbach's Conjecture, 2

In this formal statement $\varphi_{\text{even}}(x)$ is an abbreviation for something like

$$(\exists w)(x = w + w),$$

Which I would read as "There exists w such that x equals w plus w". $\varphi_{\text{prime}}(x)$ is an abbreviation for something like

$$(x > 1) \wedge (\forall u)((\exists v)(x = uv) \rightarrow ((u = 1) \vee (u = x))),$$

which I would read as "x is greater that 1 and for all u, if there exists v such that $x = uv$, then u equals 1 or x".

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One purpose for expressing Goldbach's Conjecture formally is to make the underlying structure of the sentence clear.