

## The Axioms of Set Theory.

### Equality

- (1) (Extensionality) Two sets are equal if they have the same elements.

### Existence of Special Sets

- (2) (Empty Set) There is a set with no elements.
- (3) (Infinity) There is an inductive set.

### Creation of New Sets

- (4) (Pairing) Given sets  $A$  and  $B$ , there is a set whose only elements are  $A$  and  $B$ . (I.e., the unordered pair  $\{A, B\}$  is a set.)
- (5) (Union) Given a set  $A$ , the collection of all elements of elements of  $A$  is a set. It is denoted  $\bigcup A$  and called the union of  $A$ .
- (6) (Power Set) Given a set  $A$ , the collection of subsets of  $A$  is a set. It is denoted  $\mathcal{P}(A)$  and called the power set of  $A$ .
- (7) (Separation) Given a set  $A$  and a property  $P$  given by a formula, there is a set whose elements are exactly those elements of  $A$  that satisfy  $P$ . ( $\{x \in A \mid P(x)\}$  is a set.)
- (8) (Replacement) Given a set  $A$  and a function  $F$  given by a formula, then  $\{F(x) \mid x \in A\}$  is a set.
- (9) (Choice) Given  $A = \{X_i \mid i \in I\}$ , a set of nonempty pairwise-disjoint sets, there is a set  $C$  that intersects each  $X_i$  in exactly one element.

### Sets have Special Properties

- (10) (Foundation) If  $A$  is a nonempty set, then there is an  $x \in A$  such that  $x$  and  $A$  are disjoint.  $x$  is called an  $\in$ -minimal element of  $A$ .