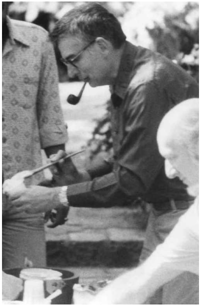
When $1 < I(T, \aleph_0) < \aleph_0$



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Ehrenfeucht's Example shows that there are complete theories with $n = 3, 4, 5, \ldots$ many isomorphism types of countable models.

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Reformulated Conjecture. If T is a complete theory in a countable language and all type spaces $S_n(T)$ are scattered, then T has countably many isomorphism types of countable models.

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