Types: Review of the definitions

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Spaces of types

Complete types of L in the variables $\mathbf{x}$ correspond to complete $L \cup \{\mathbf{c}\}$-theories, so we can import everything we learned about spaces of complete theories to speak about spaces of complete types. We write $S_n(T)$ for the space of complete n-types of T.

More detail:

1. The points of $S_n(T)$ are the complete n-types of T.
2. A basic open set of $S_n(T)$ is a set of the form
$$O_{\varphi(\mathbf{x})} = \{p \in S_n(T) \mid \varphi(\mathbf{x}) \in p\}.$$
3. In this topology, a point of the space $S_n(T)$ is topologically isolated if and only if it is isolated/supported in the sense of the previous slide. Namely, a type $p \in S_n(T)$ is isolated if there is a formula $\varphi(\mathbf{x}) \in p$ such that
$$p = \{\theta(\mathbf{x}) \mid T \models (\forall x)(\varphi(\mathbf{x}) \to \theta(\mathbf{x}))\}.$$

Realizing a type

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(A version of) Proposition 4.1.3 of Marker.

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You can realize any set of types in an elementary extension of $\mathbf{A}$ with this kind of argument.